

## Modular hyperbolas

Igor E. Shparlinski

Received: 3 September 2011 / Revised: 1 May 2012 / Accepted: 9 May 2012

Published online: 17 November 2012

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Communicated by: Takeshi Saito

**Abstract.** We give a survey of a variety of recent results about the distribution and some geometric properties of points  $(x, y)$  on modular hyperbolas  $xy \equiv a \pmod{m}$ . We also outline a very diverse range of applications of such results, discuss multivariate generalisations and suggest a number of open problems of different levels of difficulty.

*Keywords and phrases:* modular hyperbola, congruences, exponential sums, character sums

*Mathematics Subject Classification (2010):* 11D79, 11L07, 11L40, 11N69

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I.E. SHPARLINSKI

Department of Computing, Macquarie University, Sydney, NSW 2109, Australia  
(e-mail: igor.shparlinski@mq.edu.au)