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Serre's conjecture and its consequences*

Chandrashekhar Khare**

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Abstract. After some generalities about the absolute Galois group of $\mathbb{Q}$, we present the historical context in which Serre made his modularity conjecture. This was recently proved by Wintenberger and the author ([22], [23]), with an input of Kisin ([24]). The focus of these notes is on the applications of the conjecture. Some of the applications are based on the methods used in the proof.

We stress some applications to base change and descent for classical modular forms. For base change we have no new results, aside from a fragment for odd A_5-extensions of $\mathbb{Q}$. Descent for Hilbert modular forms to $\mathbb{Q}$ on the other hand is a simple corollary of Serre's conjecture.

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C. KHARE
Department of Mathematics, UCLA, 6363 Math. Sci. Bldg., 520 Portola Plaza, Los Angeles, CA 90095-1555 USA
(e-mail: shekhar@math.ucla.edu)

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