

# Invariant or quasi-invariant probability measures for infinite dimensional groups\*

## Part I: Non-ergodicity of Euler hydrodynamic

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*To the Japanese Mathematical Community,  
with my admiration and my warmest friendship.*

**Abstract.** Deterministic Euler flow on a torus cannot leave invariant any probability measure.

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