

On the structure of W -algebras in type A

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Abstract. We formulate and prove examples of a conjecture which describes the W -algebras in type A as successive quantum Hamiltonian reductions of affine vertex algebras associated with several hook-type nilpotent orbits. This implies that the affine coset subalgebras of hook-type W -algebras are building blocks of the W -algebras in type A . In the rational case, it turns out that the building blocks for the simple quotients are provided by the minimal series of the regular W -algebras. In contrast, they are provided by singlet-type extensions of W -algebras at collapsing levels which are irrational. In the latter case, several new sporadic isomorphisms between different W -algebras are established.

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