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On some number-theoretic conjectures of V. Arnold

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Abstract. In [1], V.I. Arnold conjectured "the matrix Euler congruence": $\mathrm{tr}A^{p^n} \equiv \mathrm{tr}A^{p^{n-1}} \pmod{p^n}$ for any integer matrix A, prime p, and natural number n. He proved it for $p \leq 5$, $n \leq 4$. In fact the conjecture immediately follows from a result of C.J. Smyth [5]. We give a simple proof of this result and discuss a related conjecture of Arnold concerning some congruences for multinomial coefficients.

Keywords and phrases: Euler congruences, algebraic integers, multiinomial coefficients

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