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Mean field games*

Jean-Michel Lasry** · **Pierre-Louis Lions****

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Abstract. We survey here some recent studies concerning what we call mean-field models by analogy with Statistical Mechanics and Physics. More precisely, we present three examples of our mean-field approach to modelling in Economics and Finance (or other related subjects...). Roughly speaking, we are concerned with situations that involve a very large number of "rational players" with a limited information (or visibility) on the "game". Each player chooses his optimal strategy in view of the global (or macroscopic) informations that are available to him and that result from the actions of all players. In the three examples we mention here, we derive a mean-field problem which consists in nonlinear differential equations. These equations are of a new type and our main goal here is to study them and establish their links with various fields of Analysis. We show in particular that these nonlinear problems are essentially well-posed problems i.e., have unique solutions. In addition, we give various limiting cases, examples and possible extensions. And we mention many open problems.

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J.-M. LASRY
Institut de Finance, Université Paris-Dauphine, Place du Maréchal de Lattre de Tassigny, 75775 Paris Cedex 16, France

P.-L. LIONS
Collège de France, 3 rue d'Ulm, 75005 Paris and Ceremade-UMR CNRS 7549, Université Paris-Dauphine, Place du Maréchal de Lattre de Tassigny, 75775 Paris Cedex 16, France
(e-mail: lions@ceremade.dauphine.fr)

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