

On the Connes–Kasparov isomorphism, II

The Vogan classification of essential components in the tempered dual

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Received: 17 May 2022 / Revised: 24 February 2023, 28 June 2023 / Accepted: 5 July 2023

Published online: 9 February 2024

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Communicated by: Toshiyuki Kobayashi

Abstract. This is the second of two papers dedicated to the computation of the reduced C^* -algebra of a connected, linear, real reductive group up to C^* -algebraic Morita equivalence, and the verification of the Connes–Kasparov conjecture in operator K -theory for these groups. In Part I we presented the Morita equivalence and the Connes–Kasparov morphism. In this part we shall compute the morphism using David Vogan’s description of the tempered dual. In fact we shall go further by giving a complete representation-theoretic description and parametrization, in Vogan’s terms, of the essential components of the tempered dual, which carry the K -theory of the tempered dual.

Keywords and phrases: Connes–Kasparov isomorphism, reduced C^* -algebra, real reductive group, K -theory, tempered dual, lowest K -type

Mathematics Subject Classification (2020): 22E47, 22E45, 22D25, 46L80

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