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On the Connes–Kasparov isomorphism, I

The reduced C*-algebra of a real reductive group and the *K*-theory of the tempered dual

Pierre Clare · Nigel Higson · Yanli Song · Xiang Tang

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Abstract. This is the first of two papers dedicated to the detailed determination of the reduced C*-algebra of a connected, linear, real reductive group up to Morita equivalence, and a new and very explicit proof of the Connes–Kasparov conjecture for these groups using representation theory. In this part we shall give details of the C*-algebraic Morita equivalence and then explain how the Connes–Kasparov morphism in operator K-theory may be computed using what we call the Matching Theorem, which is a purely representation-theoretic result. We shall prove our Matching Theorem in the sequel, and indeed go further by giving a simple, direct construction of the components

P. Clare
College of William & Mary, P.O. Box 8795 Williamsburg, VA 23187-8795, USA
(e-mail: prclare@wm.edu)

N. Higson
Penn State University, University Park, PA 16802, USA
(e-mail: higson@psu.edu)

Y. Song
Washington University in St. Louis, Campus Box 1089, One Brookings Drive, St. Louis, MO 63130-4899, USA
(e-mail: yanlisong@wustl.edu)

X. Tang
Washington University in St. Louis, Campus Box 1089, One Brookings Drive, St. Louis, MO 63130-4899, USA
(e-mail: xtang@math.wustl.edu)

of the tempered dual that have non-trivial K-theory using David Vogan's approach to the classification of the tempered dual.

Keywords and phrases: Connes–Kasparov isomorphism, reduced C*-algebra, real reductive group, K-theory, tempered dual

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