

Old and new challenges in Hadamard spaces

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Abstract. Hadamard spaces have traditionally played important roles in geometry and geometric group theory. More recently, they have additionally turned out to be a suitable framework for convex analysis, optimization and non-linear probability theory. The attractiveness of these emerging subject fields stems, inter alia, from the fact that some of the new results have already found their applications both in mathematics and outside. Most remarkably, a gradient flow theorem in Hadamard spaces was used to attack a conjecture of Donaldson in Kähler geometry. Other areas of applications include metric geometry and minimization of submodular functions on modular lattices. There have been also applications into computational phylogenetics and image processing.

We survey recent developments in Hadamard space analysis and optimization with the intention to advertise various open problems in the area. We also point out several fallacies in the existing proofs.

Keywords and phrases: Bi-Lipschitz embedding, convex function, gradient flow, Hadamard space, harmonic mapping, Mosco convergence, non-positive curvature, proximal mapping, proximal point algorithm, strongly continuous semigroup, submodular function, weak convergence

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