

## Three examples of residual pathologies

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**Abstract.** Several counterexamples in analysis show the existence of some special object with some sort of pathological behavior. We present three different examples where the pathological behavior is not an isolated exception, but it is the “typical” behavior of the “generic” object in a suitable class, where here generic means residual in the sense of Baire category.

The first example is the revisitation of a classical result concerning approximate differentiation. The second example is the derivative loss for solutions to linear wave equations with time-dependent Hölder continuous propagation speed. The third result is the derivative loss for solutions to transport equations with non-Lipschitz velocity field.

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