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Transgressions of the Euler class and Eisenstein cohomology of $GL_N(\mathbf{Z})$[*]

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Abstract. These notes were written to be distributed to the audience of the first author's Takagi Lectures delivered June 23, 2018. These are based on a work-in-progress that is part of a collaborative project that also involves Akshay Venkatesh.

In this work-in-progress we give a new construction of some Eisenstein classes for $GL_N(\mathbf{Z})$ that were first considered by Nori [41] and Sczech [44]. The starting point of this construction is a theorem of Sullivan on the vanishing of the Euler class of $SL_N(\mathbf{Z})$ vector bundles and the explicit transgression of this Euler class by Bismut and Cheeger. Their proof indeed produces a universal form that can be thought of as a kernel for a *regularized theta lift* for the reductive dual pair (GL_N, GL_1). This suggests looking to reductive dual pairs (GL_N, GL_k) with $k \geq 1$ for possible generalizations of the Eisenstein cocycle. This leads to fascinating lifts that relate the geometry/topology world of real arithmetic locally symmetric spaces to the arithmetic world of modular forms.

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In these notes we do not deal with the most general cases and put a lot of emphasis on various examples that are often classical.