

# Hankel transform, Langlands functoriality and functional equation of automorphic $L$ -functions<sup>★</sup>

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*Dedicated to the memory of Prof. Hiroshi Saito, with affection*

**Abstract** This is a survey on recent works of Langlands's work on functoriality conjectures and related works including the works of Braverman and Kazhdan on the functional equation of automorphic  $L$ -functions. Efforts have been made to carry out in complete generality the construction of the  $L$ -monoid, and certain a kernel which is, we believe, related to the elusive Hankel kernel.

*Keywords and phrases:* Langlands functoriality, Hankel transform, functional equation

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