

The twin prime conjecture^{*}

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Abstract. The Twin Prime Conjecture asserts that there should be infinitely many pairs of primes which differ by 2. Unfortunately this long-standing conjecture remains open, but recently there has been several dramatic developments making partial progress. We survey the key ideas behind proofs of bounded gaps between primes (due to Zhang, Tao and the author) and developments on Chowla's conjecture (due to Matomäki, Radziwiłł and Tao).

Keywords and phrases: twin prime conjecture, primes, sieves, Chowla

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