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# Mackey's theory of $\tau$-conjugate representations for finite groups

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*Dedicated to our mentors and friends*
*Toni Machì on his 75th birthday and Pierre de la Harpe on his 70th birthday*

**Abstract.** The aim of the present paper is to expose two contributions of Mackey, together with a more recent result of Kawanaka and Matsuyama, generalized by Bump and Ginzburg, on the representation theory of a finite group equipped with an involutory anti-automorphism (e.g. the anti-automorphism $g \mapsto g^{-1}$). Mackey's first contribution is a detailed version of the so-called Gelfand criterion for weakly symmetric Gelfand pairs. Mackey's second contribution is a characterization of simply reducible groups (a notion introduced by Wigner). The other result is a twisted version of the Frobenius–Schur theorem, where "twisted" refers to the above-mentioned involutory anti-automorphism.

*Keywords and phrases:* representation theory of finite groups, Gelfand pair, Kronecker product, simply reducible group, Clifford groups, Frobenius–Schur theorem

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