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On the matricial version of Fermat–Euler congruences

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Abstract. The congruences modulo the primary numbers $n = p^a$ are studied for the traces of the matrices A^n and $A^{n-\varphi(n)}$, where A is an integer matrix and $\varphi(n)$ is the number of residues modulo n, relatively prime to n.

We present an algorithm to decide whether these congruences hold for all the integer matrices A, when the prime number p is fixed. The algorithm is explicitly applied for many values of p, and the congruences are thus proved, for instance, for all the primes $p \leq 7$ (being untrue for the non-primary modulus $n = 6$).

We prove many auxiliary congruences and formulate many conjectures and problems, which can be used independently.

Keywords and phrases: Young diagram, Newton–Girard formula, multinomial coefficients, Cesaro averaging, symmetric functions, finite Lobachevsky plane, Vieta mapping, Euler zeta function, Euler group, little Fermat Theorem, geometric progression, arithmetical turbulence

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