Analytic Representation Theory of Lie Groups

Kavli IPMU
July 1 (Wed)–July 4 (Sat), 2015

PROGRAM

July 1 (Wed)

11:00–12:00	Bent Ørsted *Generalized Fourier transforms* 1. *Coxeter groups and Dunkl operators*
13:00–14:00	Yoshiki Oshima (大島芳樹)
Break	
14:15–15:15	Hisayosi Matumoto (松本久義) *Homomorphisms between scalar generalized Verma modules of* $\mathfrak{gl}(n, \mathbb{C})$
15:15–16:00	Coffee Break (Fujiwara Hall)
16:00–17:00	Takeshi Hirai (平井 武) *A review of my work on characters of semisimple Lie groups*
Break	
17:15–18:15	Kyo Nishiyama (西山 享) *Double flag variety over reals: Hermitian symmetric case*

July 2 (Thu)

13:00–14:00 Bent Ørsted
Generalized Fourier transforms
2. *Holomorphic semigroups generalizing the Hermite semigroup*

Break

14:15–15:15 Hiroyuki Ochiai (落合啓之)
Covariant differential operators and Heckman–Opdam hypergeometric systems

15:15–16:00 Coffee Break (Fujiwara Hall)

16:00–17:00 Takeshi Hirai (平井 武)
A review of my work on characters of semisimple Lie groups

Break

17:15–18:15 Masatoshi Kitagawa (北川宜稔)
On the irreducibility of $U(\mathfrak{g})^H$-modules

July 3 (Fri)

9:30–10:30 Anatoly Vershik
Representtions of current groups and theory of special representations

11:00–12:00 Toshiyuki Kobayashi (小林俊行)
Analysis of minimal representatinons
— "geometric quantization" of minimal nilpotent orbits

13:00–14:00 Bent Ørsted
Generalized Fourier transforms
3. *Generalized Fourier transforms and some applications*

14:15–15:15 Gabriele Bianchi
The covariogram and Fourier–Laplace transform in $\mathbb{C}^n$

15:15–16:00 Coffee Break (Fujiwara Hall)

16:00–17:00 Michael Pevzner
Symmetry breaking operators and resonance phenomena for branching laws

Break

17:15–18:15 Koichi Kaizuka (貝塚公一)
Scattering theory for invariant differential operators on symmetric spaces of noncompact type and its application to unitary representations

Conference Dinner (18:30 Bus)

July 4 (Sat)

9:30–10:30 Toshihisa Kubo (久保利久)
On the reducible points for scalar generalized Verma modules

11:00–12:00 Toshiyuki Kobayashi (小林俊行)
Analysis of minimal representatinons
— "geometric quantization" of minimal nilpotent orbits

13:00–14:00 Anatoly Vershik
Representtions of current groups and theory of special representations

14:15–15:15 Gabriele Bianchi
The covariogram and Fourier–Laplace transform in $\mathbb{C}^n$

15:30–16:30 Bent Ørsted
Generalized Fourier transforms
4. *Hecke algebras and interpolations between minimal representations*

ABSTRACT

Speaker: **Gabriele Bianchi** (Università di Firenze)

Title: *The covariogram and Fourier–Laplace transform in* $\mathbb{C}^n$

Abstract: The covariogram g_K of a convex body K in R^n is the function which associates to each x in R^n the volume of the intersection of K with $K + x$. Determining K from the knowledge of g_K is known as the covariogram problem. It is equivalent to determining the characteristic function 1_K of K from the modulus of its Fourier–Laplace transform, a particular instance of the phase retrieval problem. We will present this problem and a recent result that shows that when K is sufficiently smooth and in any dimension n, K is determined by g_K in the class of sufficiently smooth bodies. The proof uses in an essential way a study of the asymptotic behavior at infinity of the zero set of the Fourier–Laplace transform of 1_K in C^n done by Toshiyuki Kobayashi.
We also discuss the relevance for the covariogram problem of known determination results for the phase retrieval problem and the difficulty of finding explicit geometric conditions on K which grant that the entire Fourier–Laplace transform of 1_K cannot be factored as the product of non-trivial entire functions. This shows a connection between the covariogram problem and the Pompeiu problem.

Speaker: **Takeshi Hirai (平井 武)** (Kyoto University)

Title: *A review of my work on characters of semisimple Lie groups*

Abstract: Concentrating on the subject s of characters of semisimple Lie groups, I try to review series of my papers. The talk will contain subjects such as

- calculation of irreducible characters for $SO_0(p, 1)$ and $U(p, 1)$,
- invariant eigendistributions on semisimple Lie groups and patching conditions,
- explicit form of discrete series characters.

Speaker: **Koichi Kaizuka (貝塚公一)** (Gakushuin University)

Title: *Scattering theory for invariant differential operators on symmetric spaces of noncompact type and its application to unitary representations*

Abstract: We develop the scattering theory for invariant differential operators on symmetric spaces of noncompact type. We study asymptotic behavior of (joint) eigenfunctions in a suitable Banach space. By the scattering theory, we present three types of unitary representations of semisimple Lie groups in an explicit form as a uniform limit of representations on the Banach space.

Speaker: **Masatoshi Kitagawa (北川宜稔)** (Univ of Tokyo)

Title: *On the irreducibility of $U(\mathfrak{g})^H$-modules*

Abstract: I will report on the irreducibility of $U(\mathfrak{g})^H$-modules arising from branching problems. It is well-known that a $U(\mathfrak{g})^K$-module $\mathrm{Hom}_K(W, V)$ is irreducible for any irreducible $(\mathfrak{g}, K)$-module V and K-type W. For a non-compact subgroup H, the same statement is not true in general. In this talk, I will introduce a positive example and negative example for the irreducibility of $\mathrm{Hom}_H(W, V)$.

Speaker: **Toshiyuki Kobayashi (小林俊行)** (Univ of Tokyo)

Title: *Analysis of minimal representatinons— "geometric quantization" of minimal nilpotent orbits*

Minimal representations are the smallest infinite dimensional unitary representations of reductive groups.

About ten years ago, I suggested a program of "geometric analysis" with minimal representations as a motif.

We have found various geometric realizations of minimal representations that interact with conformal geometry, conservative quantities of PDEs, holomorphic model (e.g. Fock-type model), L^2-model (Schrödinger-type model), and Dolbeault cohomology models. I plan to discuss some of these models based on works with my collaborators, Hilgert, Mano, Möllers, and Ørsted among others.

From the viewpoint of the orbit philosoply by Kirillov–Kostant, minimal representations may be thought of as a quantization of minimal nilpotent orbits. In certain setting, we give a "geometric quantization"of minimal representations by using certain Lagrangean manifolds. Our construction includes the Schrödinger model of the Segal–Shale–Weil representation of the metaplectic group, and the commutative model of the complementary series representations of O(n,1) due to A. M. Vershik and M. I. Graev.

References

[1] T. Kobayashi. Special functions in minimal representations. In Perspectives in Representation Theory in honor of Igor Frenkel on his 60th birthday, Comtemporary Mathematics 610, 253–266. Amer. Math. Soc., Providence, RI, 2014.

[2] T. Kobayashi. Algebraic analysis of minimal representations. Publ. RIMS, 47 (2011), 585–611.

Speaker: **Toshihisa Kubo (久保利久)** (Univ of Tokyo)

Title: *On the reducible points for scalar generalized Verma modules*

Abstract: In 1980's Enright–Howe–Wallach and Jakobsen individually classified the reducible points for scalar generalized Verma modules induced from parabolic subalgebras with abelian nilpotent radicals, for which the generalized Verma modules are unitarizable. Recently, Haian He classified all the reducible points for such scalar generalized Verma modules. In this talk we will discuss about classifying reducible points for scalar generalized Verma modules induced from maximal parabolic subalgebras with two-step nilpotent radicals. This is a joint work in progress with Haian He and Roger Zierau.

Speaker: **Hisayosi Matumoto (松本久義)** (the University of Tokyo)

Title: *Homomorphisms between scalar generalized Verma modules of* $\mathfrak{gl}(n, \mathbb{C})$

Abstract: An induced module of a complex reductive Lie algebra from a one-dimensional representation of a parabolic subalgebra is called a scalar generalized Verma module. In this talk, we give a classification of homomorphisms between scalar generalized Verma modules of $\mathfrak{gl}(n, \mathbb{C})$. In fact such homomorphisms are compositions of elementary homomorphisms.

Speaker: **Kyo Nishiyama (西山 享)** (Aoyama Gakuin University)

Title: *Double flag variety over reals: Hermitian symmetric case*

Abstract: Let G be a reductive Lie group and L its symmetric subgroup, i.e., L is open in G^θ for a certain involution θ. Choose parabolic subgroups $P \subset G$ and $Q \subset L$ respectively, and put $X = G/P \times L/Q$. We call X a double flag variety. L acts on X diagonally, and X is said tobe of finite type if there are only finitely many L-orbits.
In this talk, we concentrate on the pair $(G, L) = (Sp_{2n}(\mathbb{R}), GL_n(\mathbb{R}))$ and consider $X = LGrass(\mathbb{R}^{2n}) \times Grass_d(\mathbb{R}^n)$ (product of Lagrangian Grassmannian and Grassmannian of d-dimensional subspaces). This double flag variety is turned out to be of finite type and we discuss various interesting properties of X, which is not fully investigated yet. This is based on an on-going joint work with Bent Ørsted.

Speaker: **Hiroyuki Ochiai (落合啓之)** (Kyushu University)

Title: *Covariant differential operators and Heckman–Opdam hypergeometric systems*

Abstract: This is a joint work with Tomoyoshi Ibukiyama and Takako Kuzumaki. We consider holomorphic linear differential operators with constant coefficients acting on Siegel modular forms, which preserve the automorphy when restricted to a subdomain. We give a characterization of the symbols of such differential operators, and mention an explicit form in terms of hypergeometric functions with respect to root systems introduced by G. Heckman and E. Opdam.

Speaker: **Bent Ørsted** (Århus University)

Title: *Generalized Fourier transforms*

Abstract: In these lectures, based on joint work with Salem Ben Said and Toshiyuki Kobayashi, we shall define a natural family of deformations of the usual Fourier transform in Euclidian space. The main idea is to replace the standard Laplace operator by a two-parameter family of deformations in such a way, that it still is a member of a triple generating the three-dimensional simple Lie algebra. In particular we shall describe

1. Coxeter groups and Dunkl operators
2. Holomorphic semigroups generalizing the Hermite semigroup
3. Generalized Fourier transforms and some applications
4. Hecke algebras and interpolations between minimal representations.

Speaker: **Michael Pevzner** (Reims University)

Title: *Symmetry breaking operators and resonance phenomena for branching laws*

Abstract: We shall explain the fundamental role of the Gauss hypergeometric equation in the explicit realization of symmetry breaking operators for reductive pairs and the control of multiplicities of the corresponding branching laws for singular parameters.

Speaker: **Anatoly Vershik** (St. Petersburg State University)

Title: *Representtions of current groups and theory of special representations*

Abstract: In the beginning of the 70-th H. Araki gave a general scheme of the consrutruction of the representations in the Fock space (=Ito–Wiener space) of the groups of the functions with values in Lie groups. Independently in the paper (1973) Gelfand–Graev–Vershik gave the first example of the irreducible represnetations for the case of $SL(2,R)$ and he for semi-simple group of rank one ? $O(n,1), U(n,1)$. Many authors work on this direction (VGG, Ismagilov, Delorm, Guichardet et al). The main point is the cocycle of the group with value in the irreducibe faithful represention. During the last 10 year in the papers by Graev et Vershik the following progress was obtained

1) New models ("integral model", Poisson, Quasipoussin model instead of Fock model) of the representation of current group was constructed;

2) Systematic approach to the studying of cohomology in the special unitary representations; In particulary for the Iwasawa subgroup of semsimple groups like $U(p,q), O(p,q)$ and other solvable groups.

3) Recent attepmt to extend theory for nonunitary representations.

There many open problems and link with other areas.