

Title: Independence of families of ℓ -adic representations and uniform constructibility

Abstract: Let k be a number field, \bar{k} an algebraic closure of k , $\Gamma_k = \text{Gal}(\bar{k}/k)$. A family of continuous homomorphisms $\rho_\ell : \Gamma_k \rightarrow G_\ell$, indexed by prime numbers ℓ , where G_ℓ is a locally compact ℓ -adic Lie group, is said to be independent if $\rho(\Gamma_k) = \prod \rho_\ell(\Gamma_k)$, where $\rho = (\rho_\ell) : \Gamma_k \rightarrow \prod G_\ell$. Serre gave a criterion for such a family to become independent after a finite extension of k . We will explain Serre's criterion and show that it applies to families coming from the ℓ -adic cohomology (or cohomology with compact support) of schemes separated and of finite type over k . This application uses a variant of Deligne's generic constructibility theorem with uniformity in ℓ .