Workshop: Johnson homomorphisms

June 3–7, 2013
Lecture Hall on the ground floor of the Mathematical Science Building,
The University of Tokyo

Organizers: Nariya Kawazumi, Hiroaki Nakamura, Takuya Sakasai.

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Schedule

Monday, June 3

9:25–9:30 Opening Remarks

9:30–10:30 Takuya Sakasai (The University of Tokyo)
Johnson homomorphisms up to degree 6

11:00–12:00 Takao Satoh (Tokyo University of Science)
A representation theoretic approach to the Johnson cokernels I

12:00–14:40 Lunch

14:40–15:40 Andrew Putman (Rice University)
Generating the Johnson filtration

16:10–17:10 Ryoji Kasagawa (Nihon University)
On crossed homomorphisms of the groups of volume preserving diffeomorphisms of closed manifolds

Tuesday, June 4

9:30–10:30 Eri Hatakenaka (Tokyo University of Agriculture and Technology)
On the ring of Fricke characters of free groups

11:00–12:00 Naoya Enomoto (Nara Women's University)
A representation theoretic approach to the Johnson cokernels II

12:00–14:40 Lunch

14:40–15:40 Andrew Putman (Rice University)
Generators for the hyperelliptic Torelli group and the kernel of the Burau representation at $t = -1$

16:30–18:00 Tuesday Seminar on Topology (at Room 056)
Mustafa Korkmaz (Middle East Technical University)
Low-dimensional linear representations of mapping class groups

18:30– Banquet

Wednesday, June 5

9:30–10:30 Gwénaël Massuyeau (IRMA, Université de Strasbourg)
Intersection double brackets

11:00–12:00 Susumu Hirose (Tokyo University of Science) *and* Masatoshi Sato (Gifu University)
The mod 2 Johnson homomorphism and the abelianization of the level 2 mapping class group of a non-orientable surface

Thursday, June 6

9:30–10:30 Yusuke Kuno (Tsuda College)
The Goldman-Turaev Lie bialgebra and the Johnson homomorphisms

11:00–12:00 Masanori Morishita (Kyushu University)
Johnson-Kawazumi maps in non-Abelian Iwasawa theory I – Knots and Primes

12:00–14:40 Lunch

14:40–15:40 James Conant (The University of Tennessee)
Abelianizing the target of the Johnson homomorphism: Part I

16:10–17:10 Naotake Takao (RIMS, Kyoto University)
Galois obstruction in Johnson cokernel

Friday, June 7

9:30–10:30 Jean-Baptiste Meilhan (Institut Fourier, Université de Grenoble 1)
Milnor Invariants and the HOMFLYPT polynomial

11:00–12:00 Masanori Morishita (Kyushu University)
Johnson-Kawazumi maps in non-Abelian Iwasawa theory II – Pro-p Johnson maps

12:00–14:40 Lunch

14:40–15:40 James Conant (The University of Tennessee)
Abelianizing the target of the Johnson homomorphism: Part II

16:10–17:10 Hiroaki Nakamura (Okayama University)
Some observation in Johnson homomorphisms

17:10–17:15 Closing Remarks

(Last modified: May 6)

Abstract

James Conant

Abelianizing the target of the Johnson homomorphism: Part I
Abelianizing the target of the Johnson homomorphism: Part II

The Johnson homomorphism takes values in a certain Lie algebra. Using his trace map, Morita found a large quotient of the abelianization, which he conjectured was actually all of it. Guided by Morita's construction, Kassabov, Vogtmann and I constructed a more general trace map that takes values in something called "hairy graph homology," which embeds the abelianization in the first homology of the hairy graph complex. This gives us a grading by number of loops, and so the abelianization has an induced grading. Morita's part forms the first graded summand, and we showed that the degree 2 part of the abelianization is quite large and contains spaces of modular forms with various multiplicities. However we did not exactly characterize what this degree 2 piece is. In new work we calculate the degree 2 piece exactly in terms of irreducible symplectic representations, by calculating the exact image of the trace map. In particular, the $Sp(V)$ representation theory of the abelianization is related in the simplest possible way to the $GL(V)$ representation theory of hairy graph homology. I will also discuss applications to the homology of $Out(F_n)$ and related groups.

Naoya Enomoto

A representation theoretic approach to the Johnson cokernels II

This is the latter part of a series of two lectures "A representation theoretic approach to the Johnson cokernels". In my part, first, I will explain a new class of Johnson cokernels for the mapping class groups of surfaces in the stable range. This class is obtained by using Satoh's results for structures of Johnson cokernels for automorphism groups of free groups, which are explained in part I. Second, I will describe some explicit Sp-irreducible components contained in our new class. For example, we detect an Sp-irreducible component $[1^k]$ which is regarded as the "anti-Morita obstruction". Finally, I will discuss some relationships between our new class and other developments for the Johnson cokernels for the mapping class group of surfaces.

Eri Hatakenaka

On the ring of Fricke characters of free groups

This is a joint work with Takao Satoh (Tokyo University of Science). We study a descending filtration of the ring of Fricke characters of a free group consisting of ideals on which the automorphism group of the free group naturally acts. Then by using it, we define a descending filtration of the automorphism group of a free group, and investigate a relation between it and the Andreadakis-Johnson filtration.

Susumu Hirose and **Masatoshi Sato**
The mod 2 *Johnson homomorphism and the abelianization of the level* 2 *mapping class group of a non-orientable surface*

The level 2 mapping class group of a non-orientable surface is a subgroup of the mapping class group which acts trivially on the first homology group of the surface with $\mathbb{Z}/2\mathbb{Z}$ coefficient. Szepietowski introduced the finite system of generators for this group. In this talk, we determine the abelianization and minimal set of generators of this group using the mod 2 Johnson homomorphism.

Ryoji Kasagawa
On crossed homomorphisms of the groups of volume preserving diffeomorphisms of closed manifolds

We construct crossed homomorphisms of the groups of volume preserving diffeomorphisms of closed manifolds with nontrivial first cohomology groups and give their applications to the volume flux groups. Moreover we see that they descend to crossed homomorphisms of their isotopy groups and that their restrictions to Torelli groups are the first Johnson homomorphisms in the two dimensional case.

Yusuke Kuno
The Goldman-Turaev Lie bialgebra and the Johnson homomorphisms

This talk is based on a joint work with Nariya Kawazumi (The University of Tokyo). The Goldman-Turaev Lie bialgebra is a Lie bialgebra structure on the free module with basis the homotopy set of oriented loops on an oriented surface. We show that this Lie bialgebra arises naturally in study of the Johnson homomorphisms. In particular, we show that the Lie cobracket on it, called the Turaev cobracket, gives a constraint for the Johnson image.

Gwénaël Massuyeau
Intersection double brackets

We shall review Van den Bergh's theory of "double brackets" which are non-commutative analogues of Poisson structures. We shall see that intersection of curves in a surface defines a natural double bracket on the fundamental group: this results in an intrinsic description of the quasi-Poisson structures on the representation spaces of surface groups, which have been introduced by Alekseev, Kosmann-Schwarzbach & Meinrenken. Finally, we will present other situations where "intersection double brackets" appear in a natural way. (Based on joint works with Vladimir Turaev.)

Jean-Baptiste Meilhan
Milnor Invariants and the HOMFLYPT polynomial

The purpose of this talk is to present a formula relating two link invariants of rather different nature: on one hand, Milnor invariants are extracted from the lower central series of the fundamental group of the exterior, and are close relatives of Johnson homomorphisms; on the other hand, the HOMFLYPT polynomial is a quantum invariant. After briefly reviewing the definitions, we will show how Milnor invariants of a link in the 3-sphere can be represented as a combination of HOMFLYPT polynomial of knots obtained from the link by various band sum operations. This is a joint work with A. Yasuhara.

Masanori Morishita
Johnson-Kawazumi maps in non-Abelian Iwasawa theory I – Knots and Primes
Johnson-Kawazumi maps in non-Abelian Iwasawa theory II – Pro-p Johnson maps

This is the joint work with Yuji Terashima. The purpose of my talks is to introduce arithmetic analogues of the Johnson maps - due to Nariya Kawazumi - for the study of non-Abelian Iwasawa theory in number theory. The bridge between Johnson maps and non-Abelian Iwasawa theory is given by the analogies between knots and primes. So I will start with recalling the basic analogies in arithmetic topology and explain how the Johnson maps are related to the problem in non-Abelian Iwasawa theory. Then I will present a pro-p analogue of Kawazumi's theory on the Johnson maps and show their cohomological properties.

Hiroaki Nakamura
Some observation in Johnson homomorphisms

I will discuss Johnson homomorphisms in view of Grothendieck-Teichmüller theory. Starting from an overview on stable derivations of surface Lie algebras and introducing results by Ihara, Tsunogai, Takao and Hoshi-Mochizuki et.al., I would like to pose some questions and present related observation.

Andrew Putman
Generating the Johnson filtration

I will prove that for all k, there exists some G_k such that for $g \geq G_k$, the kth term of the Johnson filtration of the genus g Torelli group is generated by elements supported on subsurfaces of genus G_k. The key tools are the notions of central stability and FI-modules introduced by myself and Church-Ellenberg-Farb, respectively. This is joint work with Tom Church.

Generators for the hyperelliptic Torelli group and the kernel of the Burau representation at $t = -1$

I will prove that the hyperelliptic Torelli group is generated by Dehn twists about separating curves that are preserved by the hyperelliptic involution. This verifies a conjecture of Hain. The hyperelliptic Torelli group can be identified with the kernel of the Burau representation evaluated at $t = -1$ and also the fundamental group of the branch locus of the period mapping. One application is that each component in Torelli space of the locus of hyperelliptic curves becomes simply-connected when curves of compact type are added. This is joint work with Tara Brendle and Dan Margalit.

Takuya Sakasai
Johnson homomorphisms up to degree 6

As a warm-up for this workshop, we first recall the original definition of Johnson homomorphisms for subgroups of the mapping class group. Their fundamental properties and tools for explicit computations, which have been developed by many researchers, will be also reviewed. Then, we will discuss how to determine the rational image of Johnson homomorphisms up to degree 6. This part is a joint work with Shigeyuki Morita and Masaaki Suzuki.

Takao Satoh
A representation theoretic approach to the Johnson cokernels I

This is the former part of a series of two talks. The latter part is given by Naoya Enomoto. In this talk, we will talk about the cokernels of the Johnson homomorphisms of automorphism groups of free groups in the stable range. In particular, we describe the rational cokernels of the Johnson homomorphisms by using trace maps. We also give a generating set of the Johnson images. Furthermore, we determine the GL-module structure of the Johnson cokernels.

Naotake Takao
Galois obstruction in Johnson cokernel

The purpose of this lecture is to introduce a number-theoretic approach to study obstructions to the surjectivity of the Johnson homomorphism. In particular, the obstruction arising from the outer Galois action on the pro-l fundamental group of the projective line minus three points over the rationals will be focused on. This obstruction has been studied by Takayuki Oda, Hiroaki Nakamura, Mamoru Asada, and the lecturer. A recent celebrated theorem on the structure of the Tannakian fundamental group of the category of mixed Tate motives over the rational integers by Francis Brown made it possible to determine the size of this obstruction.

Tuesday Seminar on Topology (June 4, Room 056, 16:30–18:00)

Mustafa Korkmaz (Middle East Technical University)
Low-dimensional linear representations of mapping class groups

For a compact connected orientable surface, the mapping class group of it is defined as the group of isotopy classes of orientation-preserving self-diffeomorphisms of S which are identity on the boundary. The action of the mapping class group on the first homology of the surface gives rise to the classical $2g$-dimensional symplectic representation. The existence of a faithful linear representation of the mapping class group is still unknown. In my talk, I will show the following three results; there is no lower dimensional (complex) linear representation, up to conjugation the symplectic representation is the unique nontrivial representation in dimension $2g$, and there is no faithful linear representation of the mapping class group in dimensions up to $3g-3$. I will also discuss a few applications of these theorems, including some algebraic consequences.