

Corrigendum to: Stable twisted cohomology of the mapping class groups in the unit tangent bundle homology (Bull. Lond. Math. Soc. 56 (2024), no. 11, 3358–3381)

Nariya Kawazumi and Arthur Soulié

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Abstract

We provide here corrections of some results and proofs of [KS24].

Correction of [KS24, Th. 1.3]. The following theorem is the corrected version of [KS24, Theorem 1.3].

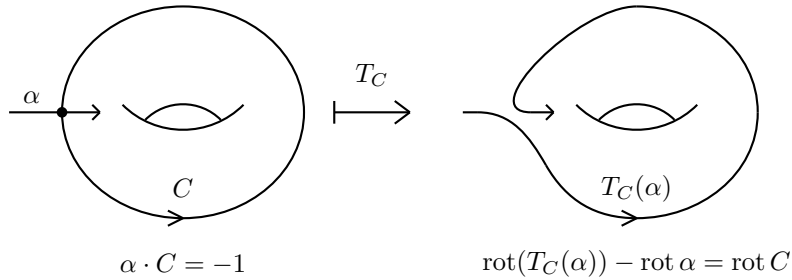
Theorem 0.1 ([Mor97, §4]) *For each $g \geq 2$, the cohomology class $k(g) := -[k_\xi(g, -)]$ does not depend on the choice of ξ and generates the infinite cyclic group $H^1(\Gamma_{g,1}; H^\vee(g))$. It is called the Earle class.*

Namely, the Earle class $k(g)$ is represented by the cocycle $-k_\xi(g, -)$ rather than by $k_\xi(g, -)$.

Correction to the proof. The first displayed equation in the proof of [KS24, Theorem 1.3] (immediately before [KS24, (1.6)]) should read

$$\text{rot}_\xi(T_C(\alpha)) - \text{rot}_\xi(\alpha) = -([\alpha] \cdot [C])\text{rot}_\xi(C).$$

This sign correction is illustrated by the following figure.



Hence [KS24, (1.6)] should be replaced by

$$k_\xi(g, T_C) = (\text{rot}_\xi(C))[C]^\vee \in H^1(\Sigma_{g,1}; \mathbb{Z}),$$

so that

$$k_\xi(g, T_{C_1}^{-1} T_{C_2}) = -(2 - 2g)[C]^\vee.$$

□

Consequences for the applications of [KS24, Th. 1.3]. Equation (2.5) in [KS24] must be corrected to

$$m_{1,1} = -k(g).$$

Indeed, it follows from (the correction of) the proof of [KS24, Th. 1.3] that the restriction of the Earle class $k(g)$ to $\pi_1(\mathrm{UT}\Sigma_{g,1})$ is equal to $-(2-2g)\mathrm{id}_H$, where we identify $\pi_1(\mathrm{UT}\Sigma_{g,1})$ with the fundamental group of the Birman kernel and id_H denotes the composite of the homomorphism induced by the projection $\mathrm{UT}\Sigma_{g,1} \rightarrow \Sigma_{g,1}$ with the identity map of H . On the other hand, [Kaw98, Assumption 4.8(1)] shows that the restriction of $m_{1,1}$ to $\pi_1(\mathrm{UT}\Sigma_{g,1})$ is equal to $(2-2g)\mathrm{id}_H$. Therefore,

$$m_{1,1} = -k(g).$$

However, this correction has no effect on the arguments and computations of [KS24, §3], and consequently all the main results of the paper remain valid.

References

- [Kaw98] N. Kawazumi. *A generalization of the Morita-Mumford classes to extended mapping class groups for surfaces*. *Invent. Math.* 131.1 (1998), pp. 137–149 (↑ 2).
- [KS24] N. Kawazumi and A. Soulié. *Stable twisted cohomology of the mapping class groups in the unit tangent bundle homology*. *Bull. Lond. Math. Soc.* 56.11 (2024), pp. 3358–3381 (↑ 1, 2).
- [Mor97] S. Morita. *Casson invariant, signature defect of framed manifolds and the secondary characteristic classes of surface bundles*. *J. Differential Geom.* 47.3 (1997), pp. 560–599 (↑ 1).

Nariya Kawazumi, *Department of Mathematical Sciences, University of Tokyo, 3-8-1 Komaba, Meguro-ku, Tokyo 153- 8914, Japan*. Email address: kawazumi@ms.u-tokyo.ac.jp

Arthur Soulié, *Normandie Univ., UNICAEN, CNRS, LMNO, 14000 Caen, France*. Email address: artsou@hotmail.fr, arthur.soulie@unicaen.fr, arthur.soulie@cnrs.fr