ON FATOU–JULIA DECOMPOSITIONS OF PSEUDOSEMIGROUPS II

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Dedicated to the 60th birthday of professor Ushiki

ABSTRACT. According to Sullivan's dictionary [9], the Julia sets for iterations of rational mappings and the limit sets of Kleinian groups are in a close relationship. In [2], we proposed a framework which unifies these dynamical systems and explains the dictionary. More concretely, notions of pseudosemigroups and their Fatou-Julia decompositions are introduced. In this article, we will introduce a result that the action of a pseudosemigroup on the Fatou set is non-expanding with respect to a Hermitian metric or a volume form on the Fatou set, and give a rough sketch of the proof. This article is an announcement of [2] and is a sequel to [3]. This is also based on a talk given at '2010 Complex Dynamics conference – Integrated Research on Complex Dynamics and its Related Fields –' held at Kyoto University.

Introduction

According to Sullivan's dictionary [9], the Julia sets for iterations of rational mappings and the limit sets of Kleinian groups are in a close relationship. The Julia sets can be also defined for transversely holomorphic foliations of complex codimension one [4], [6], [1]. Properties of the Julia sets of foliations are not yet clear, however, the Fatou sets enjoy similar properties as classical ones. In [2], we proposed a framework which unifies these dynamical systems and explains the dictionary. More concretely, notions of pseudosemigroups and their Fatou-Julia decompositions are introduced. Properties of Julia sets are still difficult to study. On the other hand, we can show under a certain condition that there exists a Hermitian metric or a volume form on the Fatou set which is 'semi-invariant' (non-expanding, and invariant under locally invertible elements. See Definition 2.2) under the action of the pseudosemigroup. In this sense, the

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action is tame on the Fatou sets. In this article, we will give a rough sketch of the proof. This article an announcement of [2] and is a sequel to [3].

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1. **Pseudosemigroups and Fatou-Julia decompositions**

We first recall notions of pseudosemigroups and their Fatou-Julia decompositions. The notion of pseudosemigroups has already appeared (cf. [8], [11] and [7]). We will make use of a similar but different one.

Definition 1.1. Let T be a topological space and Γ be a family of continuous mappings from open subsets of T into T. Then, Γ is a *pseudosemigroup* (psg for short) if the following conditions are satisfied.

1) $\mathrm{id}_T \in \Gamma$, where id_T denotes the identity map of T.
2) If $\gamma \in \Gamma$, then $\gamma|_U \in \Gamma$ for any open subset U of $\mathrm{dom}\,\gamma$.
3) If $\gamma_1, \gamma_2 \in \Gamma$ and range $\gamma_1 \subset \mathrm{dom}\,\gamma_2$, then $\gamma_2 \circ \gamma_1 \in \Gamma$, where $\mathrm{dom}\,\gamma$ and range γ denotes the domain and the range of γ, respectively.
4) Let U be an open subset of T and γ continuous mapping defined on U. If for each $x \in U$, there is an open neighborhood, say U_x, of x such that $\gamma|_{U_x}$ belongs to Γ, then $\gamma \in \Gamma$.

If in addition Γ consists of local homeomorphisms, namely, homeomorphisms from domains to ranges, then Γ is a *pseudogroup* (pg for short) if Γ satisfies 1), 2), 3) and the following conditions.

4') Let U be an open subset of T and γ a homeomorphism from U to $\gamma(U)$. If for each $x \in U$, there is an open neighborhood, say U_x, of x such that $\gamma|_{U_x}$ belongs to Γ, then $\gamma \in \Gamma$.
5) If $\gamma \in \Gamma$, then $\gamma^{-1} \in \Gamma$.

If Γ is either a psg or pg, then we set for $x \in T$

$$\Gamma_x = \{\gamma_x \mid x \in \mathrm{dom}\,\gamma\}.$$

By abuse of notation, an element of Γ_x is considered as an element of Γ defined on a neighborhood of x.

In short, a psg is a pg but the inverse is not necessarily defined. One might expect that a pg is a psg but it is not always the case.

Example 1.2. Let $T = \mathbb{C}P^1 = \mathbb{C} \cup \{\infty\}$ and define an automorphism f of $\mathbb{C}P^1$ by $f(z) = -z$. We denote by Γ the pg generated by f, that is, the smallest pg which contains f. Let $U = \{z \in \mathbb{C} \mid |z - 2| < 1\}$ and $V = f(U)$. If we set $\gamma = f|_V$, then $\gamma \cup \mathrm{id} \colon V \cup U \to U$ is not an element of Γ, because $\gamma \cup \mathrm{id}$ is not a homeomorphism. If Γ were a psg, then $\gamma \cup \mathrm{id} \in \Gamma$ by the condition 4).

Definition 1.3. Let (Γ, T) be a psg. We denote by $\mathscr{T}$ the family of relatively compact open subsets of T. If $T' \in \mathscr{T}$, then the *restriction* of Γ to T' is defined by

$$\Gamma_{T'} = \{\gamma \in \Gamma \mid \mathrm{dom}\, \gamma \subset T' \text{ and } \mathrm{range}\, \gamma \subset T'\}.$$

If (Γ, T) is *compactly generated*, then we may assume that $(\Gamma_{T'}, T')$ is *equivalent* to (Γ, T). Roughly speaking, 'compactly generated' means that the dynamics of (Γ, T) corresponds to a dynamical system on a closed manifold, and 'equivalent' means that two psg's correspond to the same dynamical system. For example, if (Γ, T) is generated by a holonomy pseudogroup of a foliation of a closed manifold, then (Γ, T) is compactly generated. We need to choose a complete transversal (transversal discs which are disjointly embedded in the ambient manifold and which meet every leaf) in order to define a holonomy pseudogroup. If we change the choice of complete transversals, then we obtain pseudogroups which are equivalent. Another source of compactly generated psg's are rational mappings on $\mathbb{C}P^1$. (Γ, T) is also compactly generated if $T = \mathbb{C}P^1$ and Γ is generated by a rational semigroup [10] acting on $\mathbb{C}P^1$. See [2] for details.

Assumption 1.4. We assume that Γ is generated by local biholomorphic diffeomorphism of $\mathbb{C}^q$, $q > 1$, or by local biholomorphic diffeomorphisms of $\mathbb{C}$ or ramified coverings, where a holomorphic map, say f, from an open set of $\mathbb{C}$ to $\mathbb{C}$ is said to be ramified covering if there exist biholomorphic diffeomorphisms

φ from dom f to a domain in $\mathbb{C}$ and ψ from range f to a domain in $\mathbb{C}$ such that $\psi \circ f \circ \varphi^{-1}(z) = z^n$ holds for some positive integer n, where $z \in$ range φ.

Note that under our assumption, Γ consists of holomorphic open mappings.

Definition 1.5. Let $T' \in \mathscr{T}$.

   1) A connected open subset U of T' is a wF-open set (weak 'Fatou'-open set) if the following conditions are satisfied:

      (a) If γ_x is the germ of an element of $\Gamma_{T'}$ at x, γ is defined on U as an element of Γ, where $(\Gamma_{T'}, T')$ is the restriction of Γ to T'.

      (b) Let Γ^U be the subset of Γ which consists of elements of Γ obtained as in (a). Then Γ^U is a normal family.

   2) A connected open subset V of T' is an F-open set ('Fatou'-open set) if $\gamma \in \Gamma'$ and if dom $\gamma \subset V$, then range γ is a union of wF-open sets.

Definition 1.6. Let (Γ, T) be a psg which fulfills Assumption 1.4. If $T' \in \mathscr{T}$, then let $F(\Gamma_{T'})$ be the union of F-open subsets of T'. Let $J(\Gamma_{T'}) = T' \setminus F(\Gamma_{T'})$, and $J_0(\Gamma) = \bigcup_{T' \in \mathscr{T}} J(\Gamma_{T'})$. Let $J(\Gamma)$ be the closure of $J_0(\Gamma)$ and $F(\Gamma) = T \setminus J(\Gamma)$. We call $F(\Gamma)$ and $J(\Gamma)$ the Fatou set and the Julia set of (Γ, T), respectively.

Roughly speaking, $J(\Gamma)$ is defined as follows. We regard $(\Gamma_{T'}, T')$ as an approximation of (Γ, T), and define $J(\Gamma_{T'})$. Indeed, it can be shown that if (Γ, T) is compactly generated, then $J(\Gamma_{T'}) = J(\Gamma) \cap T'$ holds for sufficiently large T'. If $T' \subset T''$, then $J(\Gamma_{T'}) \subset J(\Gamma_{T''}) \cap T'$ so that we take the union. Finally by taking the closure, we will obtain a set which consists of points where some 'complicated dynamics' occur in every neighborhood of that point.

Some of fundamental properties of the decomposition are as follows. First, the Julia sets and the limit sets are unified as follows.

Theorem 1.7. *If Γ is a psg, then we denote by $J_{\mathrm{psg}}(\Gamma)$ its Julia set in the sense of Definition 1.6. Then we have the following.*

   1) *If f is a rational mapping on $\mathbb{C}P^1$, then $J(f) = J_{\mathrm{psg}}(\langle f \rangle)$, where $\langle f \rangle$ denotes the pseudosemigroup generated by f. More generally, if $f_1, \ldots, f_r$ are rational mappings on $\mathbb{C}P^1$ and if G is the semigroup generated by*

$f_1, \dots, f_r$, then $J(G) = J_{\mathrm{psg}}(\langle f_1, \dots, f_r \rangle)$, where $\langle f_1, \dots, f_r \rangle$ denotes the pseudosemigroup generated by $f_1, \dots, f_r$ (or by G).

2) If f is an entire function, then let $\langle f \rangle$ be the pseudosemigroup generated by f which acts on $\mathbb{C}P^1$, where $\mathrm{dom}\, f$ is considered to be $\mathbb{C}$. Then, $J(f) \cup \{\infty\} = J_{\mathrm{psg}}(\langle f \rangle)$.

3) If G is a finitely generated Kleinian group, then $\Lambda(G) = J_{\mathrm{psg}}(\Gamma)$, where Γ is the pseudosemigroup generated by G and $\Lambda(G)$ denotes the limit set of G.

4) If Γ is the holonomy pseudogroup of a complex codimension-one foliation of a closed manifold with respect to a complete transversal. If we denote by Γ_{psg} the pseudosemigroup generated by Γ, then $J(\Gamma) = J_{\mathrm{psg}}(\Gamma_{\mathrm{psg}})$, where $J(\Gamma)$ is the Julia set of compactly generated pseudogroup in the sense of [1].

Theorem 1.7 can be seen as a refinement of Sullivan's dictionary [9].

Pseudosemigroups in Theorem 1.7 are compactly generated except the case 3). Other psg's which are not necessarily compactly generated are obtained by studying (transversely) holomorphic foliations of open manifolds, or singular holomorphic foliations. A Fatou–Julia decomposition of these foliations can be introduced by using the decomposition in the sense of Definition 1.6. In [2], some properties of such decompositions are studied.

Some of common properties of the Julia sets and the limit sets can be regarded as properties of Julia sets of compactly generated pseudosemigroups. For example, we have the following.

Lemma 1.8. *Let Γ be a compactly generated pseudosemigroup. If we denote by $F(\Gamma)$ and $J(\Gamma)$ Fatou and Julia sets of Γ, then we have the following.*

1) $F(\Gamma)$ *is forward Γ-invariant, i.e., $\Gamma(F(\Gamma)) = \Gamma$, where $\Gamma(F(\Gamma)) = \{x \in T \mid \exists\, \gamma \in \Gamma,\ \exists\, y \in F(\Gamma)\ s.t.\ x = \gamma(y)\}$.*

2) $J(\Gamma)$ *is backward Γ-invariant, i.e., $\Gamma^{-1}(J(\Gamma)) = J(\Gamma) = \{x \in T \mid \exists\, \gamma \in \Gamma,\ s.t.\ \gamma(x) \in J(\Gamma)\}$.*

2. Construction of the metric

An important property of the classical Fatou sets and the domains of discontinuity is that the action of (semi)group is tame on there. We will introduce a result which suggests that the action of psg's are also tame on the Julia sets if psg's satisfy a certain condition. In order to simplify the presentation, we will assume that Γ is generated by local biholomorphic diffeomorphisms or ramified coverings defined on open sets of $\mathbb{C}$. Under this assumption, we may assume that T is contained in $\mathbb{C}$. If (Γ, T) is a compactly generated pg, then we can construct a Hermitian metric on $F(\Gamma)$ invariant under Γ [1]. Note that this cannot remain valid for psg's. Indeed, if $z \in F(\Gamma)$, $\gamma \in \Gamma$ and $\gamma_z' = 0$, then $(\gamma^* g)_z = 0$ so that there is no Γ-invariant metric on $F(\Gamma)$, where γ_z' denotes the derivative of γ at z. For example, let $T = \mathbb{C}$ and define $f : T \to T$ by $f(z) = z^2$. Then, the open unit disc is a connected component of the Fatou set, however, f cannot be an isometry of any metric.

Inspired by the Schwarz lemma on the Poincaré disc, we introduce the notion of semi-invariant metrics as follows.

Definition 2.1. Let g_1 and g_2 be Hermitian metrics on $F(\Gamma)$. If $z \in F(\Gamma)$, then we denote by $(g_1)_z$ the metric on $T_z F(\Gamma)$. Suppose that we have $g_1 = f_1^2 g_0$ and $g_2 = f_2^2 g_0$ on a neighborhood of z, where g_0 denotes the standard Hermitian metric on $\mathbb{C}$. If $f_1(z) \leq f_2(z)$, then we write $(g_1)_z \leq (g_2)_z$. Note that this condition is independent of the choice of charts about z. If $(g_1)_z \leq (g_2)_z$ holds on $F(\Gamma)$, then we write $g_1 \leq g_2$.

Definition 2.2. Let g be a Hermitian metric on $F(\Gamma)$. The metric g is said to be *semi-invariant* if $z \in F(\Gamma)$ and if $\gamma \in \Gamma$ is defined on a neighborhood of z, then $\gamma^* g \leq g$ holds on $\operatorname{dom} \gamma$.

Let $(\Gamma_{T'}, T')$ be a restriction of (Γ, T) to $T' \in \mathscr{T}$. We may assume that $T' = \coprod_{i=1}^{r} T_i'$, where each T_i' is the unit open ball in $\mathbb{C}^q$. Let η_ϵ, $0 < \epsilon < 1$, be a smooth non-negative function on $\mathbb{R}$ such that

    1) $\eta_\epsilon(t) = 1$ on $(-\infty, 1 - \epsilon]$,
    2) η_ϵ is strictly decreasing on $[1 - \epsilon, 1]$,
    3) $\eta_\epsilon(t) = 0$ on $[1, +\infty)$.

Definition 2.3. Let z_i be the standard coordinates on T_i' and set $h_i(z_i) = \eta_\epsilon(\|z_i\|)$, where $\|\cdot\|$ denotes the standard norm on $\mathbb{C}$. The set of functions $\{h_i\}$ is denoted by h and considered as a function on T'. We will represent functions and differential forms on T' in the same way. We define a function f on T' by

$$f(z) = \sup_{\gamma \in (\Gamma')_z} |\gamma_z'| \, h(\gamma(z)),$$

where $|\gamma_z'|$ denotes the absolute value of the derivative of γ at z. Finally we set $g = f^2 g_0$.

It is easy to see that g is semi-invariant under Γ and that g is lower-semicontinuous. Let

$$\Gamma^\times = \left\{ \gamma \in \Gamma \,\middle|\, \begin{array}{l} \forall\, z \in \operatorname{dom}\gamma \;\exists\, \text{neighborhood } U_z \text{ of } z \text{ in } \operatorname{dom}\gamma, \\ \text{s.t. } \gamma|_{U_z} \text{ is a homeomorphism and } (\gamma|_{U_z})^{-1} \in \Gamma \end{array} \right\}.$$

Theorem 2.4. 1) *Suppose that (Γ, T) is compactly generated, then the metric g is finite and locally Lipschitz continuous on $F(\Gamma)$.*

  2) *If $\Gamma^\times = \Gamma$, then $F(\Gamma)$ admits a Hermitian metric which is locally Lipschitz continuous and Γ-invariant.*

  3) *If (Γ, T) is generated by a compactly generated pg, then $F(\Gamma)$ admits a Hermitian metric which is of class C^ω and Γ-semiinvariant.*

Instead of giving a proof, we will describe an idea of the construction. See [2] for details. In order to simplify the explanation, we will deal with norms. First assume that (Γ, T) is compactly generated. Then, we may assume that $(\Gamma_{T'}, T')$ is equivalent to (Γ, T), and it suffices to construct a metric on $F(\Gamma_{T'})$. Suppose that a Hermitian metric, say g_0, on $F(\Gamma)$ is given (we do not assume that g_0 is semi-invariant under Γ). Let $z \in F(\Gamma_{T'})$ and if $v \in T_z F(\Gamma_{T'})$. If we set $\|v\|_z = \sup_{\gamma \in (\Gamma_{T'})_z} \|\gamma_* v\|_{\gamma z}$, then it is semi-invariant and lower-semicontinuous. The semi-invariance is easy to see. In order to show the lower-semicontinuity, we choose $\gamma \in \Gamma_{T'}$ such that $\|\gamma_* v\|_{\gamma z}$ is close enough to $\|v\|_z$. We can make use of γ for estimating $\|v\|_w$ for $w \in F(\Gamma_{T'})$ close enough to z. When showing upper-semicontinuity, we also choose $\gamma \in \Gamma_{T'}$ which gives an approximation of $\|\cdot\|_z$ as above. Since $x \in F(\Gamma_{T'})$, we may assume that γ is well-defined on a neighborhood, say U, of z. However, the extension is an element of

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Γ so that $\gamma(U)$ needs not be contained in T'. This prevents us from using U for estimating $\|v'\|_{w'}$, where $w' \in U$ and $v' \in T_{w'}F(\Gamma_{T'})$. In order to avoid this difficulty, we modify the metric g_0 by multiplying the function η in Definition 2.3 so that $\|\gamma_* v\|_{\gamma z}$ is small if γz is close to the boundary of T'. After the modification, the fact that $\|\gamma_* v\|_{\gamma z}$ approximates $\|v\|_z$ implies that γz is contained in a compact subset of T'. Since Γ^U is a normal family, we may now assume that $\gamma(U)$ is contained in $F(\Gamma_{T'})$. Then, we can obtain a uniform estimate of $\|v'\|_w$ for $w \in U$ and $v' \in T_w F(\Gamma_{T'})$ to show the local Lipschitz continuity. If moreover (Γ, T) is generated by a compactly generated pg, then the metric is necessarily Γ-invariant. We can study a kind of the closure of Γ^U (cf. [5]) and its action on $F(\Gamma)$. By modifying arguments in [4] (and [6]), we can classify the action and show the existence of an invariant metric of class C^ω [1]. Next, we assume that (Γ, T) is not necessarily compactly generated but $\Gamma^\times = \Gamma$. In this case, we choose a sequence of restrictions (Γ_n, T_n), where $\{T_n\} \subset \mathscr{T}$ such that $T_n \Subset T_{n+1}$, $\Gamma_n = \Gamma_{T_n}$ $(n = 1, 2, \ldots)$ and $\bigcup_{n=1}^\infty T_n = T$. We can construct a Γ_1-invariant, locally Lipschitz continuous metric, say g_1, on $F(\Gamma) \cap T_1$ by a similar method as above. Since $\Gamma = \Gamma^\times$, we can extend g_1 to $F(\Gamma) \cap T_2$ by the Γ_2-action. Indeed, suppose that $z \in F(\Gamma) \cap T_1$, $w \in F(\Gamma) \cap T_2$ and $\gamma_1, \gamma_2 \in \Gamma_2$ satisfy $\gamma_1(z) = \gamma_2(z) = w$. Then, we may assume that $\gamma_2^{-1}\gamma_1 \in \Gamma_1$ so that $|\gamma_1'|_z = |\gamma_2'|_z$. Hence $(\gamma_1^{-1})^* g_{1z} = (\gamma_2^{-1})^* g_{2z}$ holds. Let h_2 be a metric on T_2 such that h_2 coincides with the extension of g_1 on $\Gamma_2(F(\Gamma) \cap T_1)$. By repeating the above construction with replacing g_0 by h_2, we can find a metric on $F(\Gamma) \cap T_2$ invariant under Γ_2. In this way, we can find an invariant metric on $F(\Gamma)$.

Example 2.5 ([2, Example 4.21]). We define $\gamma \colon \mathbb{C}P^1 \to \mathbb{C}P^1$ by $\gamma(z) = z^2$. Then, $J(\gamma) = \{|z| = 1\}$. If we set

$$
f(z) = \begin{cases} 1 & \text{if } |z| \leq \frac{1}{2}, \\ 2^k |z|^{2^k - 1} & \text{if } 2^{-\frac{1}{2^{k-1}}} \leq |z| \leq 2^{-\frac{1}{2^k}}, \\ 2^k |z|^{-2^k - 1} & \text{if } 2^{\frac{1}{2^k}} \leq |z| \leq 2^{\frac{1}{2^{k-1}}}, \\ \frac{1}{|z|^2} & \text{if } |z| \geq 2, \end{cases}
$$

then $g = f^2 |dz|^2$ gives a Hermitian metric on $\mathbb{C}P^1 \setminus \{|z| = 1\}$ which is locally Lipschitz continuous and semi-invariant under the action of Γ, where $\Gamma = \langle \gamma \rangle$. On the other hand, if we consider the Poincaré metric on the unit disc, then γ is contracting by the Schwarz lemma. Hence the Poincaré metrics on the unit disc and $\mathbb{C}P^1 \setminus \{|z| \leq 1\}$ give rise to a Hermitian metric on $\mathbb{C}P^1 \setminus \{|z| = 1\}$ which is of class C^ω and semi-invariant under the action of Γ. On the other hand, there is no Γ-invariant metric on $F(\Gamma)$. Indeed, $0 \in F(\Gamma)$ but $(\gamma^* g)_0 = 0$ for any metric g on $F(\Gamma)$.

Let $\widehat{\Gamma}$ be the psg generated by $\gamma|_{\mathbb{C}P^1 \setminus \{0, \infty\}}$ and its local inverses. Then $F(\widehat{\Gamma}) = \mathbb{C} \setminus (S^1 \cup \{0\})$. An invariant metric on $F(\widehat{\Gamma})$ is given by $|dz|^2 / (|z| \log |z|)^2$ on $\{0 < |z| < 1\}$. We can find on $\{1 < |z|\}$ a metric of the same kind.

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