LARGE DEVIATION PRINCIPLES FOR GENERALIZED FEYNMAN-KAC FUNCTIONALS AND ITS APPLICATIONS

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In this talk, we mainly focus on the large deviation theory for non-local Feynman-Kac functionals which do not necessarily admit bounded variation (namely, generalized Feynman-Kac functionals) in the framework of symmetric doubly Feller or strong Feller processes. As applications, we deduce the L^p-independence of the spectral bound of our generalized Feynman-Kac semigroup under our conditions.

Let E be a locally compact separable metric space and m a positive Radon measure on E with full topological support. Let $\mathbf{X} = (\Omega, X_t, \mathbf{P}_x, \zeta, x \in E)$ be an m-symmetric Hunt process on E and $(\mathcal{E}, \mathcal{F})$ the associated symmetric Dirichlet form on $L^2(E; m)$. We always assume that $(\mathcal{E}, \mathcal{F})$ is irreducible and $\mathbf{X}$ has doubly Feller property. For a symmetric bounded function F on $E \times E$ vanishing on the diagonal set, define the discontinuous AF $A_t^F = \sum_{0<s\le t} F(X_{s-}, X_s)$. Let $\mathcal{F}_e$ be the extended Dirichlet space of $(\mathcal{E}, \mathcal{F})$. For $u \in \mathcal{F}_e \cap C_\infty(E)$, let N^u be the CAF of zero energy in the strict sense of the Fukushima decomposition of $u(X_t) - u(X_0)$. Set an AF $A := N^u + A^{\mu,F}$ with $A^{\mu,F} := A^\mu + A^F$. Here $A_t^\mu := A_t^{\mu_+} - A_t^{\mu_-}$, and $A_t^{\mu_+}$ (resp. $A_t^{\mu_-}$) is the PCAF in the strict sense associated with μ_+ (resp. μ_-) as its Revuz measure. Let (N, H) be a Lévy system for $\mathbf{X}$ and put $N(F)(x) := \int_{E_\partial} F(x,y) N(x, dy)$. We consider the following multiplicative functional of the form:

$$e_A(t) := \exp\left(N_t^u\right) \operatorname{Exp}\left(A^{\mu,F}\right)_t, \quad t \ge 0, \tag{1}$$

where $\operatorname{Exp}(B)_t$ stands for the Stieltjes exponential of B. Define the associated Feynman-Kac semigroup by $Q_t f(x) := \mathbf{E}_x[e_A(t) f(X_t)]$ for $x \in E, f \in \mathcal{B}_+(E)$. Let $\mathcal{P}(E)$ denote the space of all Borel probability measures on E. Define a rate function $I_{\mathcal{Q}}(\nu)$ on $\mathcal{P}(E)$ by

$$I_{\mathcal{Q}}(\nu) := \begin{cases} \mathcal{Q}(\phi, \phi) & \text{if } \nu \ll m \text{ and } \phi := \sqrt{d\nu/dm} \in \mathcal{D}(\mathcal{Q}) \\ +\infty & \text{otherwise} \end{cases}$$

Here $\mathcal{Q}(f, g) := \mathcal{E}(f, g) + \mathcal{E}(u, fg) - \mathcal{H}(f, g)$ with

$$\mathcal{H}(f, g) := \int_E f(x) g(x) \mu(dx) + \iint_{E \times E \setminus d} f(x) g(y) F(x, y) N(x, dy) \mu_H(dx).$$

For $\omega \in \Omega$ with $t < \zeta(\omega)$, consider the following normalized occupation time distribution $L_t(\omega) \in \mathcal{P}(E)$ by

$$L_t(\omega)(A) := \frac{1}{t} \int_0^t \mathbf{1}_A(X_s(\omega)) ds \quad \text{for } A \in \mathcal{B}(E).$$

Theorem 1. *Suppose $\mu_{\langle u \rangle} \in S_K^1$ (Kato class in the strict sense), $\mu = \mu_+ - \mu_-$ and $F = F_+ - F_-$ with $\mu_+ + N(F_+)\mu_H \in S_{LK}^1 \cap S_{EK}^1$ (local and extended Kato classes in the strict sense), $\mu_- + N(F_-)\mu_H \in S_{LK}^1$.*

(i) *For any open set* $G \subset \mathcal{P}(E)$ *and* $x \in E$,

$$\underline{\lim}_{t\to\infty} \frac{1}{t} \log \mathbf{E}_x[e_A(t) : L_t \in G, t < \zeta] \geq - \inf_{\nu\in G} I_{\mathcal{Q}}(\nu). \tag{2}$$

(ii) *Assume* $\mu_- + N(F_-)\mu_H \in S^1_{LK} \cap S^1_D$ *(local Kato and Dynkin classes in the strict sense). Then for any compact set* $K \subset \mathcal{P}(E)$,

$$\overline{\lim}_{t\to\infty} \frac{1}{t} \log \sup_{x\in E} \mathbf{E}_x[e_A(t) : L_t \in K, t < \zeta] \leq - \inf_{\nu\in K} I_{\mathcal{Q}}(\nu). \tag{3}$$

(iii) *Assume further* $m \in S^1_{K^+_\infty}$ *(positive order Green-tight Kato class in the strict sense) and* $\mu_- + N(F_-)\mu_H \in S^1_{LK} \cap S^1_D$. *Then for any closed set* $K \subset \mathcal{P}(E)$, *we have* (3). *In particular,*

$$\lim_{t\to\infty} \frac{1}{t} \log \mathbf{E}_x[e_A(t) : t < \zeta] = \lim_{t\to\infty} \frac{1}{t} \log \sup_{x\in E} \mathbf{E}_x[e_A(t) : t < \zeta] = - \inf_{\nu\in\mathcal{P}(E)} I_{\mathcal{Q}}(\nu). \tag{4}$$

We use the convention that $F = F_+ - F_- \in J^1_* - J^1_{**}$ means $N(F)\mu_H = N(F_+)\mu_H - N(F_-)\mu_H \in S^1_* - S^1_{**}$. If we assume $\mu_{\langle u\rangle} \in S^1_K$, $\mu_\pm \in S^1_K$ and $F_\pm \in J^1_K$, then we can obtain the same conclusions as in Theorems 1 without assuming the Feller property of $\mathbf{X}$. For $p \in [0,\infty]$, let $\lambda_p(u,\mu,F)$ be the L^p-spectral radius of our Feynman-Kac semigroup $\{Q_t\}_{t>0}$.

Theorem 2. *Suppose* $\mu_{\langle u\rangle} \in S^1_{K^+_\infty}$, $\mu = \mu_+ - \mu_- \in S^1_{K^+_\infty} - S^1_{LK} \cap S^1_D$ *and* $F = F_+ - F_- \in J^1_{K^+_\infty} - J^1_{LK} \cap J^1_D$. *Then the spectrum radius* $\lambda_p(u,\mu,F)$ $(1 \leq p \leq \infty)$ *is independent of* p *if* $\lambda_2(u,\mu,F) \leq 0$. *Moreover, suppose that* $\mathbf{X}$ *is conservative,* $\mu_- \in S^1_{K^+_\infty}$ *and* $F_- \in J^1_{K^+_\infty}$. *Then* $\lambda_2(u,\mu,F) > 0$ *implies* $\lambda_\infty(u,\mu,F) = 0$.

Corollary 1. *Suppose* $\mu_{\langle u\rangle} \in S^1_{K^+_\infty}$. *Assume* $\mu = \mu_+ - \mu_-$ *with* $\mu_+ \in S^1_{K^+_\infty}$, $\mu_- = 0$, *and* $F = F_+ - F_-$ *with* $F_+ \in J^1_{K^+_\infty}$, $F_- = 0$. *Then* $\lambda_2(0,0,0) \leq 0$ *implies* $\lambda_2(u,\mu,F) \leq 0$, *in particular,* $\lambda_p(u,\mu,F)$ $(1 \leq p \leq \infty)$ *is independent of* p *if* $\lambda_2(0,0,0) \leq 0$. *Moreover, if* $\mathbf{X}$ *is transient,* $\mu_{\langle u\rangle} \in S^1_{K^+_\infty}$, $\mu = \mu_+ - \mu_- \in S^1_{K^+_\infty} - S^1_{K_\infty}$ *and* $F = F_+ - F_- \in J^1_{K^+_\infty} - J^1_{K_\infty}$ *then the same conclusion holds. Here* $S^1_{K_\infty}$ *denotes the* 0*-order Green-tight Kato class in the strict sense.*

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