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A quantum Frobenius map and tensor product theorem for cyclotomic q-Schur algebras

Abstract: A cyclotomic q-Schur algebra $\mathscr{S}_{n,r}$ is a quasi-hereditary cover of an Ariki-Koike algebra (cyclotmic Hecke algebra) associated with a complex reflection group of type G(r,1,n). In the case where $r = 1$, $\mathscr{S}_{n,r}$ is a quotient of the quantum group $U_q(\mathfrak{gl}_m)$ associated with the general linear lie algebra $\mathfrak{gl}_m$. In the case where $r \geq 2$, $\mathscr{S}_{n,r}$ is not a quotient of a quantum group. However, $\mathscr{S}_{n,r}$ has the upper (resp. lower) Borel subalgebra which is a quotient of the upper (resp. lower) Borel subalgebra of $U_q(\mathfrak{gl}_m)$. Motivated by this fact, the speaker introduced a certain algebra $\mathcal{U}$ associated with the Cartan data of $\mathfrak{gl}_m$, and realized $\mathscr{S}_{n,r}$ as a quotient of $\mathcal{U}$. The algebra $\mathcal{U}$ contains the Cartan subalgebra of $U_q(\mathfrak{gl}_m)$, and we can develop the highest weight theory to study the representations of $\mathscr{S}_{n,r}$.

On the other hand, there exists an functor from $\mathscr{S}_{n,r}$-mod to $\mathcal{O}_{\mathfrak{g}}^{(n)}$, where $\mathfrak{g} = \mathfrak{gl}_{m_1} \oplus \cdots \oplus \mathfrak{gl}_{m_r}$ is a Levi subalgebra of $\mathfrak{gl}_m$, and $\mathcal{O}_{\mathfrak{g}}^{(n)}$ is a full subcategory of $U_q(\mathfrak{g})$-mod consisting of polynomial representations of degree n. In the case of special parameters, this functor gives an equivalence of categories. Then, representations of $\mathscr{S}_{n,r}$ is related to $U_q(\mathfrak{g})$ rather than $U_q(\mathfrak{gl}_m)$.

In this talk, when q is a root of unity, we give a quantum Frobenius map from $\mathcal{U}$ to the universal enveloping algebra $U(\mathfrak{g})$ of $\mathfrak{g}$, and prove that a simple $\mathscr{S}_{n,r}$-module with a certain highest weight killed by the Frobenius kernel. We also give an analogue of Steinberg's tensor product theorem for simple modules of cyclotomic q-Schur algebras. As an application, we give a (weak) categorification of the Heisenberg action on a (level r) Fock space.