GROUP ACTIONS ON INJECTIVE FACTORS

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Abstract. We survey recent progress in classification of group actions on injective factors. We deal with three kinds of groups; discrete groups with Kazhdan's property T, the real number group, and compact abelian groups. For the first and second groups, actions on the injective factor of type II_1 are considered, and for the last one, we give a complete classification on injective factors of type III.

1. Introduction

Since Connes' breakthrough[9,11] for classification of integer actions on the injective factor of type II_1, several authors have studied classification problem of group actions up to (cocycle/stable) conjugacy on injective factors. As Connes' original research was tightly connected to structure of injective factors of type III_λ, $0 < \lambda < 1$, automorphism approach for studying von Neumann algebras has been one of the most powerful and fruitful ones. Here we present recent progress in this area. We deal with three topics; actions of discrete groups with Kazhdan's property T on the injective factor $\mathcal{R}$ of type II_1,[25] one-parameter automorphism groups of $\mathcal{R}$,[21,22,23,24] and compact abelian group actions on injective factors of type III.[26,27]

Jones[16] and Ocneanu[29] followed Connes and extended classification to finite and discrete amenable group actions on the injective factor $\mathcal{R}$ of type II_1, respectively.

The most important result in their work is that an outer action of a discrete amenable group G on $\mathcal{R}$ is unique up to cocycle conjugacy. Here cocycle conjugacy means that in addition to conjugacy, we allow a perturbation of an action α by a unitary cocycle u_g with $u_{gh} = u_g\alpha_g(u_h), g, h \in G$. Later, Jones showed that this uniqueness characterizes discrete amenable groups[17,18]; if a discrete group G is non-amenable, then it has two mutually non-cocycle conjugate outer actions on $\mathcal{R}$. He used ergodicity on the central sequence algebra to distinguish two actions. Because discrete groups with Kazhdan's property T[20,43] are known to be opposite extremes of amenable groups, we expect that the contrast also holds in group actions. We show that this is the case by showing that 1-cohomology for actions of groups with Kazhdan's property T "almost" vanishes under a certain condition. This approach also characterizes Kazhdan's property T in terms of cohomology of actions on factors of type II_1. These will be explained in Section 1.

Section 2 is devoted to one-parameter automorphism groups. Compact abelian group actions on injective factors of type II are classified by Jones-Takesaki[19] by reducing the problem to discrete abelian group actions on semifinite injective von Neumann algebras, which are not necessarily factors, via Takesaki duality.[39] Now the most important group, for still unsolved classification problem of actions on the injective factor of type II_1, is the real number group $\mathbf{R}$. Study of one-parameter automorphism groups of injective factors of type II is related to that of injective factor of type III_1, as in the relation of automorphisms of injective factors of type II and injective factors of type III_λ, $0 < \lambda < 1$. Though uniqueness of the injective factor of type III_1 was finally established by Connes and Haagerup,[13,15] classification of one-parameter automorphism groups of injective factors of type II would deepen the understanding of its structure. For example, as Connes' method[9] is useful for Loi's study of subfactors of injective factors of type III_λ, $0 < \lambda < 1$,[28] it would give much information for study of subfactors of the injective factor of type III_1. Though complete classification has not been obtained, certain progress for several interesting cases has been made. This will be presented in Section 2.

We show a complete classification of compact abelian group actions on injective factors of type III in Section 3. This is a joint work with M. Takesaki. Sutherland and Takesaki classified discrete amenable group actions on injective factors of type III_λ, $0 < \lambda < 1$,[38] and these two and the author classified discrete *abelian* group actions on injective factors of type III_1.[26] Since abelian groups are amenable, we should be able to use these results to obtain a classification of compact abelian group actions on injective factors of type III via Takesaki duality as in work of Jones-Takesaki for semifinite cases.[19] However two kinds of new difficulty arise in type III situation. One is caused by the fact that each fibre in the central decomposition of an injective von Neumann algebra of type III is not necessarily mutually isomorphic even when we have a centrally ergodic action on it. The other arises from the difference of centrally trivial automorphisms and inner automorphisms. We show in Section 3 that the difficulties can be overcome. Because the classification of Sutherland, Takesaki and the author for discrete groups is up to cocycle conjugacy, we get a classification up to stable conjugacy at first. (Two actions are said to be stably conjugate if their second dual actions are conjugate.) Stable conjugacy is weaker than cocycle conjugacy, but the two notions coincide when the factor is properly infinite, hence we get a cocycle conjugacy classification for injective factors of type III, and after it, we get a complete classification (up to conjugacy) by means of inner invariant as in Jones-Takesaki.[19]

2. Actions of groups with Kazhdan's property T

Significance of Kazhdan's property T[20] in theory of operator algebras was first pointed out by Connes[12]. He showed a group von Neumann algebra of a discrete ICC group with Kazhdan's property T has a countable fundamental group by rigidity argument. Since then, several authors have studied property T in operator algebra situations and in ergodic theory.[35,42,43] Roughly speaking, rigidity argument shows two mathematical objects are "equivalent" if they are "close" in a certain sense.

We would like to apply this type of argument to unitary cocycles. Our basic observation is that we have the following correspondence in the study of automorphism groups and unitary cocycles:

$$\begin{aligned} \text{Automorphism} &\longleftrightarrow \text{Cocycle} \\ \text{Inner automorphism} &\longleftrightarrow \text{Coboundary} \\ \text{Factor} &\longleftrightarrow \text{Ergodic action} \end{aligned}$$

Thus for an ergodic action α of a discrete group with Kazhdan's property T on a factor of type II_1, we consider α-unitary cocycles u_g with $\mathrm{Ad}(u_g)\cdot\alpha_g$ is also ergodic, then the number of cohomology classes of these can be shown to be at most countable. This means the number of unitary cocycles connecting ergodic actions is "small". Thus if we have a continuous family of mutually non-conjugate outer ergodic actions on the injective factor $\mathcal{R}$ of type II_1, then their cocycle conjugacy classes have the cardinality continuum, which is the opposite extreme of uniqueness of outer actions for discrete amenable groups. Here note that the possible maximal cardinality of cocycle conjugacy classes is always continuum. (See

Popa's articles[31,32] for related results and problems.) Groups $SL(n, \mathbf{Z})$, $n \geq 3$, and $Sp(n, \mathbf{Z})$, $n \geq 2$, are typical ones with Kazhdan's property T, and M. Choda[5,6] showed that these have a continuous family of outer ergodic actions on $\mathcal{R}$. (Her construction uses the irrational rotation algebra A_θ for different θ.) Thus we get the following.[25]

Theorem. *Each of $SL(n, \mathbf{Z})$, $n \geq 3$, and $Sp(n, \mathbf{Z})$, $n \geq 2$, has a continuous family of mutually non-cocycle conjugate ergodic outer actions on the injective factor of type II_1.*

Each of these actions has ergodicity at infinity in the sense of Jones[18] and the crossed product algebra by each action has property T as shown by M. Choda.[6] Thus these actions cannot distinguished by these properties.

It may seem artificial that we used only cocycles u_g with $\mathrm{Ad}(u_g)\cdot\alpha_g$ ergodic in the above argument. But without this restriction, the conclusion is invalid, and these cocycles make a closed subset in the set of all the cocycles. This trouble comes from non-commutativity of the algebra, which prevents unitary cocycles from making a group. These would justify the above restriction. We can also show the fixed point algebra of $\mathrm{Ad}(u_g) \cdot \alpha_g$ continuously depends on u_g as follows.[25]

Proposition. *Let α be an action of a discrete group G with Kazhdan's property T on a factor $\mathcal{M}$ of type II_1. Then the correspondence from a unitary α-cocycle u_g to the fixed point algebra $\mathcal{M}^{Ad(u_g)\cdot\alpha_g}$ is uniformly continuous in the following sense: For any $\varepsilon_0 > 0$, there exists $\delta > 0$ such that if $d(u_g, v_g) < \delta$, then $\|\mathcal{M}^{Ad(u_g)\cdot\alpha_g} - \mathcal{M}^{Ad(v_g)\cdot\alpha_g}\|_2 < \varepsilon_0$.*

See Christensen[7] for a notation. Here the metric d is defined by $d(u_g, v_g) = \sum_{n=1}^{\infty} \dfrac{1}{2^n}\|u_{g_n} - v_{g_n}\|_2$. This shows another opposite extreme of amenable group actions. (Ocneanu's theorem[29] shows that an ergodic outer action of discrete amenable groups on $\mathcal{R}$ can be perturbed to an outer action with a large fixed point algebra with an arbitrarily small cocycle.)

This kind of cohomological consideration also characterizes Kazhdan's property T, which extends a result of Araki and M. Choda[2] as follows.[25]

Theorem. *Let G ba a discrete group. Then the following conditions are equivalent.*

(1) *G has Kazhdan's property T.*
(2) *Any action of G on a factor of type II_1 is strong in the sense of Araki-Choda.*[2]

(3) *For any action α of G on a factor of type II_1, each cohomology class is closed.*
(4) *For any action α of G on a factor of type II_1, the 1-cohomology space with the quotient topology is Hausdorff.*

Amenability can be also characterized in terms of cohomological terms of group actions in a similar way to Schmidt's result[34] as follows.[25]

Theorem. *Let G be a countable group. Then the following conditions are equivalent.*

(1) *G is amenable.*
(2) *No free action of G on the injective factor $\mathcal{R}$ of type II_1 is strong in the sense of Araki-Choda.*[2]
(3) *For all outer actions of G on the injective factor $\mathcal{R}$ of type II_1, coboundaries make a dense subset of the set of all the cocycles, but these two sets do not coincide.*

3. One-parameter automorphism groups

First recall that classification of one-parameter automorphism groups of injective factors of type II is related to classification of injective factor of type III_1 in the sense that uniqueness of the injective factor of type III_1 is equivalent to uniqueness of a one-parameter automorphism group α of the injective factor of type II_∞ with $\mathrm{tr}\cdot\alpha_t = e^{-t}\mathrm{tr}$. (See Takesaki.[39]) Thus classification of trace-preserving cases is still open. We present a recent progress in classification of one-parameter automorphism groups of the injective factor of type II_1 here, though the complete classification has not been obtained yet.

All the work of Connes, Jones and Ocneanu[9,16,29] showed that a free action of a certain group on the injective factor $\mathcal{R}$ of type II_1 is unique up to cocycle conjugacy. In this context, a free action means an action α for which α_g is outer for all $g \neq 1$. Then what does a "free" action mean for the real number group ? It is easy to see that for a one-parameter automorphism α_t, the condition that α_t is outer for all $t \neq 0$ does not imply that the crossed product algebra is a factor. This is an analogue of the fact that $T(\mathcal{M}) = \{0\}$ does not imply $S(\mathcal{M}) = [0, \infty)$ for a factor $\mathcal{M}$ of type III. (See Connes[7] for T-sets and S-sets.) Hence we use Connes spectrum $\Gamma(\alpha)$ as an analogue of S-sets. But it turns out that unlike type III factor cases, $\Gamma(\alpha) = \mathbf{R}$ does not imply that α_t is outer for all $t \neq 0$,[21] thus in order to get a uniqueness result we would like to assume both $\Gamma(\alpha) = \mathbf{R}$, which is equivalent to saying that the crossed product algebra is a factor, and $\alpha_t \notin \mathrm{Int}(\mathcal{R})$ for all $t \neq 0$. We remark that these two are implied by $(\mathcal{R} \rtimes_\alpha \mathbf{R}) \cap \mathcal{R}' = \mathbf{C}$. (Note that if $\Gamma(\alpha) \neq \mathbf{R}$, a one-parameter automorphism group α can be classified more easily.[21,22])

A splitting of a model action from a given free action was a key step in work of all of Connes, Jones, and Ocneanu. We next discuss what the model action is for the real number group. Ocneanu's model[29] is obtained by an infinite tensor product of an almost left regular representations on subsets of the group. But we use an analogue of modular automorphism groups for one-parameter automorphism groups of $\mathcal{R}$, and choose the following as the model action.

$$\bigotimes_{n=1}^{\infty} \mathrm{Ad}\exp it \begin{pmatrix} -\lambda/2 & 0 \\ 0 & \lambda/2 \end{pmatrix} \otimes \mathrm{Ad}\exp it \begin{pmatrix} -\mu/2 & 0 \\ 0 & \mu/2 \end{pmatrix}.$$

Here λ and μ are non-zero real numbers with $\lambda/\mu \notin \mathbf{Q}$. This is an analogue of the ITPFI factor of type III_1 of Araki-Woods,[3] and we can show the cocycle conjugacy class of this action does not depend on the choice of λ and μ, and moreover an infinite tensor product type actions of any size of matrix gives the same cocycle conjugacy class if it has full Connes spectrum $\mathbf{R}$ as follows.[22]

Theorem. *If a one-parameter automorphism group α of the injective factor $\mathcal{R}$ of type II_1 fixes a Cartan subalgebra of $\mathcal{R}$ elementwise and $\Gamma(\alpha) = \mathbf{R}$, then its cocycle conjugacy class is unique.*

This is an analogue of uniqueness of the ITPFI (or Krieger) factor of type III_1.[3] This theorem is also valid for any locally compact abelian separable group action.

Our approach to the splitting problem of a model action from a "free" one-parameter automorphism α of the injective factor $\mathcal{R}$ of type II_1 is as follows.

Approach. *Find a weakly dense C^*-algebra A of $\mathcal{R}$ on which α is a norm continuous action, and a norm-dense $*$-subalgebra A^∞ of C^∞ elements in A with respect to α and C^*-norm with the following properties:*

(1) For any $\varepsilon > 0$, there exists a 2×2 matrix unit (e_{jk}) of $A_{e_{11}+e_{22}}$ in A^∞ with $tr(1 - e_{11} - e_{22}) < \varepsilon$.

(2) For a given $\nu \in \mathbf{R}$ and $x_1, \ldots, x_n \in A^\infty$ and a matrix unit (e_{jk}) in (1), there exists an automorphism σ of A^∞ such that $\|\delta(\sigma(e_{jk})) - i|j-k|\nu e_{jk}\| < \varepsilon$ and $\|[\sigma(e_{jk}), x_l]\| < \varepsilon$, where δ is the derivation generating α.

In general, it is too much that we expect existence of a matrix unit in A, because of K-theoretic restrictions. Thus we adopt a weaker version (1) as above. The second estimate in (2) is for making a central sequence, and related to asymptotical abelian systems.[35] The first estimate in (2) is for making a piece of the model action. Condition (2) is similar to Araki's property L'_λ,[1] and a key tool for producing the model action as Ocneanu's non-commutative Rohlin theorem.[29] By working on several estimates using (1), (2), we can show a splitting of the model action in terms of central sequences.[23]

But unfortunately, it is unknown how to produce A^∞ and σ in a general setting, and we are lead to working on a concrete example in detail. A key to the uniqueness result cited above was abundant existence of elements fixed by α. From this point, the opposite extreme is an ergodic action. Now we deal with the following construction of an action.

In the irrational rotation C^*-algebra A_θ with $uv = e^{2\pi i\theta} vu$, consider the following one-parameter automorphism group α_t: $\alpha_t(u) = e^{i\lambda t}u, \alpha_t(v) = e^{i\mu t}v$. Here λ and μ are non-zero real numbers with $\lambda/\mu \notin \mathbf{Q}$. We extend this one-parameter automorphism group to the weak closure $\mathcal{R}$ of A_θ with respect to the trace τ, which is the injective II_1 factor. It is easy to see this action has full Connes spectrum $\mathbf{R}$ but this is outer if and only if λ/μ is not in the $GL(2, \mathbf{Q})$ orbit of θ, where a $GL(2, \mathbf{Q})$-action is given by a fractional transformation. This subtlety is another interesting point of this action. We would like to show that this action is cocycle conjugate to the model action. Though this actions is almost periodic (i.e., this action can be extended to a $\mathbf{T}^2$ action), it is impossible to perturb this $\mathbf{T}^2$ action to a product type one, by Olesen-Pedersen-Takesaki.[30] It makes this action the best one for trying our approach. For this action, we use a Rieffel projection construction[33] for (1) in the above approach and an $SL(2, \mathbf{Z})$ action on A_θ considered by Brenken[4] and Watatani[41] for (2) in the approach. Then a delicate estimate shows the condition $\lambda/\mu \notin GL(2, \mathbf{Q})\theta$ exactly implies (2). Hence we get a splitting of the model action. Then we can use the following theorem to show the model action "absorbs" our action α.[23]

Theorem. *Let α be an almost periodic one-parameter automorphism group of $\mathcal{R}$. Let β_t be an infinite tensor product type one-parameter automorphism group of $\mathcal{R}$ with $\Gamma(\beta) = \mathbf{R}$. Then $\alpha_t \otimes \beta_t$ is cocycle conjugate to the β_t on $\mathcal{R}$.*

Hence we get the following.[23]

Theorem. *Let α be the one-parameter automorphism as above. We assume $\lambda/\mu \notin \mathbf{Q}$, and λ/μ is not in the $GL(2,\mathbf{Q})$ orbit of θ. Then α_t is cocycle conjugate to an infinite tensor product type action β of $\mathbf{R}$ on $\mathcal{R}$ with $\Gamma(\beta) = \mathbf{R}$, which is unique up to cocycle conjugacy.*

Certainly it is an open interesting problem how far our approach can go in general.

4. Compact abelian group actions

After Sutherland and Takesaki classified discrete amenable group actions on injective factors of type III_λ, $0 \le \lambda < 1$, up to cocycle conjugacy,[38] they and the author extended the same type of classification to discrete abelian group actions on the injective factor of type III_1.[26] In the both work, we have to invoke a result of Ocneanu[29] at a certain stage. Because the classes of approximately inner automorphisms and centrally trivial automorphisms play an important role in Ocneanu's theorem as in Connes' work,[9] we need characterizations for these two classes of automorphisms on injective factors of type III. Connes announced these characterizations in 1976[10] as follows, but the proof has not been available for a long time, though this theorem has been used in Connes[13] and Sutherland-Takesaki.[38]

Theorem. *For injective factors $\mathcal{M}$ of type III, we have:*

(i) *$Ker(mod) = \overline{Int}(\mathcal{M})$;*
(ii) *An automorphism α of $\mathcal{M}$ is centrally trivial if and only if α is of the form $\alpha = Ad(u)\cdot\bar\sigma^\varphi_c$, where $\bar\sigma^\psi_c$ is an extended modular automorphism for a dominant weight φ on $\mathcal{M}$, c is a θ-cocycle on $\mathcal{U}(\mathcal{C}_\varphi)$, and $u \in \mathcal{U}(\mathcal{M})$.*

(See Connes-Takesaki[14] for related definitions and notations.)

Sutherland, Takesaki and the author supplied a proof of the above theorem for the first time,[26] and made use of the characterizations. After Ocneanu's work,[29] Jones and Takesaki[19] classified compact abelian group actions on injective factors of type II via Takesaki duality. They work on dual actions, which no more act on factors in general, and regard the actions as groupoid actions on factors. It is now natural to expect the same approach extends in type III settings. We show that this is the case, but several kinds of technical difficulty arise from the fact that the algebra is of type III.[27]

For a given action α of a separable compact abelian group A on an injective factor $\mathcal{M}$ of type III, we work on its dual action on $\mathcal{M} \rtimes_\alpha A$. We would like to represent the crossed product algebra as $\mathcal{N} \bar\otimes L^\infty(X,\mu)$ for a certain injective factor $\mathcal{N}$ and work on a groupoid $X \rtimes \hat{A}$ to use tools like Krieger's cohomology lemma as in Jones-Takesaki.[19] But this type of central decomposition is impossible in general because classification of type III_0 factors is not smooth. That is, each fibre factor in the central decomposition is not mutually isomorphic in general. This phenomenon actually occurs in this crossed product algebra setting as shown in our work.[27] Hence we have to take a more technical version of groupoid approach, and will use Sutherland's generalized cohomology lemma[36] instead of Krieger's. Then we can apply the groupoid approach of Sutherland-Takesaki.[37] The next difficulty arises from the fact that the class of centrally trivial automorphisms is now strictly larger than the class of inner automorphisms. For example, we have to prove that in the central decomposition $\mathcal{M} \rtimes_\alpha A = \int_X^\oplus \mathcal{N}(x)\, d\mu(x)$, the correspondence $x \mapsto \mathrm{Cnt}(\mathcal{N}(x))$ is Borel in a certain sense defined by Sutherland.[36] This can be done since we have a complete characterization of $\mathrm{Cnt}(\mathcal{N}(x))$.[26]

An outline of the arguments is as follows. First, consider two extreme cases. In one case, we have a factor crossed product algebra $\mathcal{M} \rtimes_\alpha A$. This type of action is called prime, and we have a full Connes spectrum, $\Gamma(\alpha) = \hat{A}$, in this case. Then we have invariants as in Sutherland-Takesaki[38] for the dual action on the factor $\mathcal{M} \rtimes_\alpha A$. In the other extreme case, we have a zero Connes spectrum, $\Gamma(\alpha) = \{0\}$. Then we have an (ergodic) faithful action of $\hat{A}$ on the center $\mathcal{Z}(\mathcal{M} \rtimes_\alpha A)$ and the module given by the dual action, as invariants. (The action of $\hat{A}$ is like a flow of weights, and the module is also a part of invariants in the first case.)

In general, the situation is a mixture of these two, but no new invariants appear, and these are all we need. The first family of invariants are handles by a classification of Sutherland-Takesaki,[38] and these two and the author.[26] We use Sutherland's generalized cohomology lemma[36] to deal with the second family of invariants. For the cohomology lemma, we need a certain density argument. For example, in the easiest situation, we need $\mathrm{Ker}\,(\mathrm{mod}) = \overline{\mathrm{Int}}$, and here we need more delicate result. The following is a key to it.

Theorem. *Let $\mathcal{M}$ be an injective factor of type III, and φ be a dominant weight on $\mathcal{M}$. If an automorphism α of $\mathcal{M}$ is approximately inner, then there exist a unitary $u \in \mathcal{M}$ and a sequence of unitaries $\{v_n\}$ in $\mathcal{M}_\varphi$ such that $\alpha = Ad(u) \cdot \lim_{n\to\infty} Ad(v_n)$.*

Here inner perturbation by u does not matter very much, thus this essentially means that approximately inner automorphisms are approximated by unitaries in the centralizer. For the proof of the above theorem, we need some technical lemmas, but we have a complete list of automorphisms of injective factors of type III,[26,38] hence it is enough to check the conclusion for all the members of the list.

Then we have to combine results for $\Gamma(\alpha)$ and $\hat{A}/\Gamma(\alpha)$ to supply a complete cocycle. This can be done by constructing a groupoid model action on the injective factor $\mathcal{R}$ of type II_1 which can kill any bicharacter on the groupoid times a subgroup of $\hat{A}$. Because all the fibre factors in the central decomposition split $\mathcal{R}$ as a tensor product factor, this is meaningful. Then the error from being a cocycle is given by a certain bicharacter, and it can be killed by a good property of the model action. The final form is as follows.[27]

Theorem. *Let α and β be actions of a compact abelian group A on an injective factor $\mathcal{M}$ of type III. Then we conclude:*

i) α and β are cocycle conjugate if and only if

a) $\mathcal{M} \rtimes_\alpha A \cong \mathcal{M} \rtimes_\beta A$,

b) $N(\hat{\alpha}) = N(\hat{\beta})$,

c) there exists an isomorphism θ of $\mathcal{F}(\mathcal{M} \rtimes_\alpha A)$ onto $\mathcal{F}(\mathcal{M} \rtimes_\beta A)$ which conjugates the restriction of $\hat{\alpha}$ and $\hat{\beta}$ to the center $\mathcal{Z}(\mathcal{M} \rtimes_\alpha A)$ and $\mathcal{Z}(\mathcal{M} \rtimes_\beta A)$ and $\theta(mod(\hat{\alpha}), \chi_{\hat{\alpha}}, \nu_{\hat{\alpha}}) = (mod(\hat{\beta}), \chi_{\hat{\beta}}, \nu_{\hat{\beta}})$.

ii) α and β are conjugate if and only if we have $\iota(\alpha) = \iota(\beta)$ in addition to the above three conditions.

Here $N(\hat{\alpha})$ is the set of $g \in \hat{A}$ such that $\hat{\alpha}_g$ fixes the center of the crossed product algebra $\mathcal{M} \rtimes_\alpha A$ and each restriction $\hat{\alpha}_{g,x}$ to each fibre is centrally trivial. The notation $\mathcal{F}$ denotes the flow of weights,[14] χ, ν denote the characteristic invariants and modular invariants of a discrete group actions on injective factors of type III,[38] and ι denote the inner invariant of Jones-Takesaki.[19]

For prime actions with properly infinite fixed point algebras, our results says that Thomsen's classification[40] extends, while Thomsen proved the result under the assumption that the fixed point algebra is of type II_∞.

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