EXTENSIONS OF AMENABLE C*-ALGEBRAS WITH GIVEN SPECTRUM

EBERHARD KIRCHBERG (HU BERLIN, U OF COPENHAGEN)

1. Summary/Abstract:

I'll discuss the following recent theorem and its corollaries. (I mentioned the theorem more than a year ago as a conjecture/question. Here the name "Dini space" denotes the second countable, locally compact and sober T_0 spaces X, despite Dini spaces have its own natural definition.)

Theorem 1.1 (EK). *Let X a Dini space, and suppose that U is a dense open subset of X, and that there are stable, amenable and separable C*-algebras $A \cong A \otimes \mathcal{O}_2$ and $B \cong B \otimes \mathcal{O}_2$ and homeomorphisms h_B from $\mathrm{Prim}(B)$ onto U and h_A from $\mathrm{Prim}(A)$ onto $F = X \setminus U$.*

Then there exists a unique (up to unitary equivalence) Busby invariant $\beta\colon A \to \mathrm{Q}(B) := \mathcal{M}(B)/B$, such that $\mathrm{Prim}(E)$ is homeomorphic to X (in a natural way) for the corresponding extension

$$0 \to B \to E \to A \to 0\,,$$

with $E := (\pi_B)^{-1}\beta(A)$, where $\pi_B\colon \mathcal{M}(B) \to \mathrm{Q}(B)$ is the natural epimorphism.

Corollary 1.2 (O.Ioffe,EK). *All coherent Dini spaces X are homeomorphic to primitive ideal spaces $\mathrm{Prim}(A)$ of amenable and separable C*-algebras A.*

The "natural" structure on $\mathrm{Prim}(E)$ is given by by a (unique) homeomorphism $\gamma\colon X \to \mathrm{Prim}(E)$, such that $\gamma \circ h_B$ becomes the identity map on $\mathrm{Prim}(B)$, and $\gamma \circ h_A$ is the homeomorphism from $\mathrm{Prim}(A)$ onto $\mathrm{Prim}(E/B)$ induced by the Busby invariant β. The requirement, that this γ^{-1} defines the same topology on $X = U \cup F$ can be rephrased by transformation conditions:

$$k(\gamma(\overline{G})) = E \cap \mathcal{M}(B, k((h_B)^{-1}G))$$

and

$$\beta(A) \cap \pi_B(\mathcal{M}(B, k((h_B)^{-1}G))) = \beta(k((h_A)^{-1}(F \cap \overline{G}))).$$

for every (relatively) closed subset G of U and closures $\overline{G}$ in X.