KK-theory and Conformal Field Theory (and also reconstruction)

Sendai, 12 August 2016

Terry Gannon (U Alberta)

joint work David Evans (Cardiff U)

Conformal field theory (CFT)

Recall: CFT is conformally invariant quantum field theory on
2-dimensional space-time

- Full CFT contains theory of holomorphic quantum fields (a chiral CFT).

Conformal field theory (CFT)

Recall: CFT is conformally invariant quantum field theory on 2-dimensional space-time

- Full CFT contains theory of holomorphic quantum fields (a chiral CFT).
- Full CFT contains theory of anti-holomorphic quantum fields (also a chiral CFT).

Conformal field theory (CFT)

Recall: CFT is conformally invariant quantum field theory on 2-dimensional space-time

- Full CFT contains theory of holomorphic quantum fields (a chiral CFT).
- Full CFT contains theory of anti-holomorphic quantum fields (also a chiral CFT).
- Full CFT can be recovered by splicing together those two chiral halves.

Conformal field theory (CFT)

Recall: CFT is conformally invariant quantum field theory on 2-dimensional space-time

- Full CFT contains theory of holomorphic quantum fields (a chiral CFT).
- Full CFT contains theory of anti-holomorphic quantum fields (also a chiral CFT).
- Full CFT can be recovered by splicing together those two chiral halves.
- Understanding how the full theory is recovered from the chiral halves is theme of much of Fuchs–Runkel–Schweigert and collaborators

Chiral conformal field theory

The two main mathematical approaches to chiral CFT are:

Vertex operator algebras (VOAs): Wightman axioms.

Conformal nets of factors: Haag–Kastler axioms.

These should be more or less equivalent: see Carpi–Kawahigashi–Longo–Weiner, arXiv:1503.01260.

Rational conformal field theory

All-important are representations of chiral CFT.
Simplest case: semi-simple representation theory.
These CFT are called rational.
Corresponding VOAs are called strongly-rational.
Corresponding conformal nets are called completely-rational.
Expect a bijection between unitary strongly-rational VOAs, and completely-rational conformal nets.

Examples:

- one associated to even positive-definite lattices L: simple modules in bijection with cosets L^*/L

Rational conformal field theory

All-important are representations of chiral CFT.
Simplest case: semi-simple representation theory.
These CFT are called rational.
Corresponding VOAs are called strongly-rational.
Corresponding conformal nets are called completely-rational.
Expect a bijection between unitary strongly-rational VOAs, and completely-rational conformal nets.

Examples:

- one associated to even positive-definite lattices L: simple modules in bijection with cosets L^*/L
- one associated to affine sl(2) at each positive integer level k: simple module for each highest-weight $0, 1, ..., k$

Modular tensor categories

The category of representations of strongly-rational VOAs and completely-rational conformal nets are modular tensor categories (MTC).
These are braided semi-simple tensor categories, with duals.
The braiding is nondegenerate=maximally nonsymmetric.

Get finite-dimensional representations of all surface mapping class groups (e.g. $\mathrm{SL}_2(\mathbb{Z})$).
Infinitely many different VOAs and conformal nets will have the same MTC.

Reconstruction

Conjecture. Every unitary modular tensor category is the category of representations of a strongly-rational VOA and completely-rational conformal net.

It is an analogue of Tannaka–Krein duality: groups can be recovered from their category of representations

Circumstantial evidence only for this conjecture.

Simple fact: $\mathrm{MTC}(\mathcal{V}_1 \otimes \mathcal{V}_2) =$ Deligne product of $\mathrm{MTC}(\mathcal{V}_1)$ and $\mathrm{MTC}(\mathcal{V}_2)$

However: Galois associate of MTC is MTC; no known analogue of Galois associate of VOA.

Holomorphic VOA/holomorphic conformal net

A VOA or conformal net corresponding to trivial MTC is called holomorphic.

E.g. a theory $\mathcal{V}(L)$ corresponding to self-dual even positive-definite lattice L.

The 'smallest' holomorphic theories are:

central charge $c = 8$: $\mathcal{V}(E_8)$;
central charge $c = 16$: $\mathcal{V}(E_8 \oplus E_8)$ and $\mathcal{V}(D_{16}^+)$;
central charge $c = 24$: at least 71 inequivalent ones, including the Monstrous moonshine module

Quantum doubles

The easiest way to construct MTC, is as the quantum double=Drinfeld double of a fusion category.
A fusion category is semisimple tensor category with duals, not necessarily braided.

VOAs associated to quantum doubles should be precisely the VOAs contained with finite index in a holomorphic VOA.

(finite index here means branching rules are finite)

By contrast, any rational VOA should be contained in a holomorphic VOA, but with infinite index.

Sexy examples

Any finite-depth finite index subfactor defines two fusion categories. They have same double.

Examples: Haagerup subfactor, Asaeda–Haagerup, extended Haagerup, ...

The double of Haagerup subfactor seems to be contained with finite index in $\mathcal{V}(E_8)$ (Evans-G; G-Lam)

The double of extended Haagerup subfactor seems to be contained with finite index in $\mathcal{V}(E_8)$ (G-Morrison)

Doubles of finite groups

Easiest class of examples of fusion categories: $\mathrm{Vect}_\omega(G)$ where G is finite group and $\omega \in Z^3(G; \mathbb{T})$ (ω twists associativity)
Their doubles are denoted $\mathcal{D}_\omega(G)$
At least some of those doubles are realised by orbifolds $\mathcal{V}^G$ of holomorphic VOAs.

E.g. $H^3(\mathbb{Z}_n; \mathbb{T}) \cong \mathbb{Z}_n$.

- Lie group E_8 has two elements of order 2, up to conjugation. They both act as automorphisms on $\mathcal{V}(E_8)$.

Doubles of finite groups

Easiest class of examples of fusion categories: $\mathrm{Vect}_\omega(G)$ where G is finite group and $\omega \in Z^3(G;\mathbb{T})$ (ω twists associativity)
Their doubles are denoted $\mathcal{D}_\omega(G)$
At least some of those doubles are realised by orbifolds $\mathcal{V}^G$ of holomorphic VOAs.

E.g. $H^3(\mathbb{Z}_n;\mathbb{T}) \cong \mathbb{Z}_n$.

- Lie group E_8 has two elements of order 2, up to conjugation. They both act as automorphisms on $\mathcal{V}(E_8)$.
  - One gives $\mathcal{V}(D_8)$, corresponding to untwisted $\mathcal{D}_1(\mathbb{Z}_2)$.

Doubles of finite groups

Easiest class of examples of fusion categories: $\mathrm{Vect}_\omega(G)$ where G is finite group and $\omega \in Z^3(G; \mathbb{T})$ (ω twists associativity)
Their doubles are denoted $\mathcal{D}_\omega(G)$
At least some of those doubles are realised by orbifolds $\mathcal{V}^G$ of holomorphic VOAs.

E.g. $H^3(\mathbb{Z}_n; \mathbb{T}) \cong \mathbb{Z}_n$.

- Lie group E_8 has two elements of order 2, up to conjugation. They both act as automorphisms on $\mathcal{V}(E_8)$.
  - One gives $\mathcal{V}(D_8)$, corresponding to untwisted $\mathcal{D}_1(\mathbb{Z}_2)$.
  - The other gives $\mathcal{V}(A_1 \oplus E_7)$, corresponded to twisted $\mathcal{D}_\omega(\mathbb{Z}_2)$.

Doubles of finite groups

Easiest class of examples of fusion categories: $\mathrm{Vect}_\omega(G)$ where G is finite group and $\omega \in Z^3(G;\mathbb{T})$ (ω twists associativity)
Their doubles are denoted $\mathcal{D}_\omega(G)$
At least some of those doubles are realised by orbifolds $\mathcal{V}^G$ of holomorphic VOAs.

E.g. $H^3(\mathbb{Z}_n;\mathbb{T}) \cong \mathbb{Z}_n$.

- Lie group E_8 has two elements of order 2, up to conjugation. They both act as automorphisms on $\mathcal{V}(E_8)$.
  - One gives $\mathcal{V}(D_8)$, corresponding to untwisted $\mathcal{D}_1(\mathbb{Z}_2)$.
  - The other gives $\mathcal{V}(A_1 \oplus E_7)$, corresponded to twisted $\mathcal{D}_\omega(\mathbb{Z}_2)$.
- E_8 also has two elements of order 3.

Doubles of finite groups

Easiest class of examples of fusion categories: $\mathrm{Vect}_\omega(G)$ where G is finite group and $\omega \in Z^3(G;\mathbb{T})$ (ω twists associativity)
Their doubles are denoted $\mathcal{D}_\omega(G)$
At least some of those doubles are realised by orbifolds $\mathcal{V}^G$ of holomorphic VOAs.

E.g. $H^3(\mathbb{Z}_n;\mathbb{T}) \cong \mathbb{Z}_n$.

- Lie group E_8 has two elements of order 2, up to conjugation. They both act as automorphisms on $\mathcal{V}(E_8)$.
  - One gives $\mathcal{V}(D_8)$, corresponding to untwisted $\mathcal{D}_1(\mathbb{Z}_2)$.
  - The other gives $\mathcal{V}(A_1 \oplus E_7)$, corresponded to twisted $\mathcal{D}_\omega(\mathbb{Z}_2)$.
- E_8 also has two elements of order 3.
  - One gives $\mathcal{V}(A_2 \oplus E_6)$, giving untwisted $\mathcal{D}_1(\mathbb{Z}_3)$;

Doubles of finite groups

Easiest class of examples of fusion categories: $\mathrm{Vect}_\omega(G)$ where G is finite group and $\omega \in Z^3(G;\mathbb{T})$ (ω twists associativity)
Their doubles are denoted $\mathcal{D}_\omega(G)$
At least some of those doubles are realised by orbifolds $\mathcal{V}^G$ of holomorphic VOAs.

E.g. $H^3(\mathbb{Z}_n;\mathbb{T}) \cong \mathbb{Z}_n$.

- Lie group E_8 has two elements of order 2, up to conjugation. They both act as automorphisms on $\mathcal{V}(E_8)$.
  - One gives $\mathcal{V}(D_8)$, corresponding to untwisted $\mathcal{D}_1(\mathbb{Z}_2)$.
  - The other gives $\mathcal{V}(A_1 \oplus E_7)$, corresponded to twisted $\mathcal{D}_\omega(\mathbb{Z}_2)$.
- E_8 also has two elements of order 3.
  - One gives $\mathcal{V}(A_2 \oplus E_6)$, giving untwisted $\mathcal{D}_1(\mathbb{Z}_3)$;
  - One gives $\mathcal{V}(A_8)$, giving twisted $\mathcal{D}_\omega(\mathbb{Z}_3)$.

Reconstruction for finite group doubles

Theorem (Evans-G) Each twisted double $\mathcal{D}_\omega(G)$ is the category of representations for the orbifold $\mathcal{A}^G$ of some holomorphic conformal net $\mathcal{A}$ and some group of automorphisms G.

The same is expected to be true for VOAs. The ingredient missing for VOAs is the theorem that finite group orbifolds of holomorphic VOAs are rational. For conformal nets this was proved long ago by Xu (not just for holomorphic conformal nets, but completely-rational ones). For VOAs, what is known at present is that, when G is solvable and $\mathcal{V}$ is strongly-rational, then $\mathcal{V}^G$ is strongly-rational.

Module categories

This theorem is a corollary of a deeper result of ours.

Recall that a full CFT is spliced together from the two chiral halves. Each possible splicing corresponds to a module category.

This is a category which can be thought of as a module for a MTC; it is a categorical formulation of a nimrep=nonnegative integer matrix representation of the fusion ring.

Associated to a module category is a modular invariant partition function $\mathcal{Z} = \sum_{\lambda,\mu} \mathcal{Z}_{\lambda,\mu} \chi_\lambda \overline{\chi_\mu}$ which makes explicit the splicing.

The module category defines a finite-index extension of each chiral CFT=VOA=conformal net, and an equivalence of the MTC's of those extensions.

Examples: Module categories for affine algebra MTC

- The module categories for the MTC of affine sl(2) at level k, fall into an A-D-E pattern.

Examples: Module categories for affine algebra MTC

- The module categories for the MTC of affine sl(2) at level k, fall into an A-D-E pattern.
  - e.g. the D_{even} module categories correspond to a simple-current extension of both chiral CFTs;

Examples: Module categories for affine algebra MTC

- The module categories for the MTC of affine sl(2) at level k, fall into an A-D-E pattern.
  - e.g. the D_{even} module categories correspond to a simple-current extension of both chiral CFTs;
  - e.g. the D_{odd} ones correspond to a nontrivial automorphism of the underlying MTCs.

Examples: Module categories for affine algebra MTC

- The module categories for the MTC of affine sl(2) at level k, fall into an A-D-E pattern.
  - e.g. the D_{even} module categories correspond to a simple-current extension of both chiral CFTs;
  - e.g. the D_{odd} ones correspond to a nontrivial automorphism of the underlying MTCs.
- e.g. the module categories for the MTC of affine sl(3) at level k, are closely related to simple factors of Jacobians of Fermat curves.

Module categories for finite group doubles

Ostrik (2003) found an abstract nonsense parametrisation of all module categories of $\mathcal{D}_\omega(G)$:

they correspond to pairs (H, ψ), where $H \leq G \times G$ and $\psi \in Z^2(H; \mathbb{T})$.
When $\omega \neq 1$, there is a compatibility condition for H and ω.

But it was very unclear what the corresponding modular invariant $\mathcal{Z}$ is, and what the corresponding chiral extensions and automorphism is.

Not just interesting to CFT: Implicit here are all possible finite index extensions of holomorphic orbifolds.

Understanding more explicitly Ostrik's classification, is the task we (Evans–G) tried to address.

Twisted equivariant K-theory

Our starting point is a geometric interpretation of $\mathcal{D}_\omega(G)$: K-theory.
In our context, not very complicated at all, very classical.

Think of the K-group as classifying vector bundles.
Vector bundles over a point are vector spaces, which are classified by dimension, so $K(\mathrm{pt}) = \mathbb{Z}$.

When a finite group G acts on those spaces, we get the equivariant K-group $K_G(\mathrm{pt})$, which we'll also write in groupoid language as $K(\mathrm{pt}/\!/G)$.
It can be identified with character ring R_G.

We can twist it by a 3-cocycle $\omega \in Z^3(G;\mathbb{T})$, which we'll write ${}^\omega K_G(\mathrm{pt}) = K(\mathrm{pt}/\!/_\omega G)$. This will be the additive group of projective characters of G.

Twisted K-theory for finite group doubles

The geometric interpretation of the Grothendieck ring=fusion ring $\mathcal{D}_\omega(G)$ is not much more complicated:
the Grothendieck ring=fusion ring of MTC $\mathcal{D}_\omega(G)$ can be naturally identified with the twisted equivariant K-group ${}^\omega K_G(G) = K(G//_\omega G)$, where G acts on itself by conjugation.

We think of these as bundles over the 0-dimensional space G. G-equivariance means that the irreducible bundles live over each orbit (here, each conjugacy class), and carry an irrep of the stabiliser (here, the centraliser). The twist ω makes these irreps projective. Explicitly:

$$K_0^G(G) = \oplus_{conj.classes} K^0_{C_G(g)}(1) = \oplus_{conj.cl.} R_{C_G(g)}$$

This matches the simple objects of $\mathcal{D}_1(G)$ being pairs (g, χ).
In fact ${}^\omega K_G(G)$ forms a ring.

Twisted KK-theory for finite group doubles

Why should that help us understand Ostrik's classification of module categories?

- The corresponding modular invariant partition function $\mathcal{Z}$ is a matrix, with rows and columns parametrised by simple objects in the MTC $\mathcal{D}_\omega(G)$.

Twisted KK-theory for finite group doubles

Why should that help us understand Ostrik's classification of module categories?

- The corresponding modular invariant partition function $\mathcal{Z}$ is a matrix, with rows and columns parametrised by simple objects in the MTC $\mathcal{D}_\omega(G)$.
- In other words, $\mathcal{Z}$ is a linear endomorphism on its fusion ring.

Twisted KK-theory for finite group doubles

Why should that help us understand Ostrik's classification of module categories?

- The corresponding modular invariant partition function $\mathcal{Z}$ is a matrix, with rows and columns parametrised by simple objects in the MTC $\mathcal{D}_\omega(G)$.
- In other words, $\mathcal{Z}$ is a linear endomorphism on its fusion ring.
- But that fusion ring is a K-group.

Twisted KK-theory for finite group doubles

Why should that help us understand Ostrik's classification of module categories?

- The corresponding modular invariant partition function $\mathcal{Z}$ is a matrix, with rows and columns parametrised by simple objects in the MTC $\mathcal{D}_\omega(G)$.
- In other words, $\mathcal{Z}$ is a linear endomorphism on its fusion ring.
- But that fusion ring is a K-group.
- So $\mathcal{Z}$ is an element in $KK_0(G//_\omega G, G//_\omega G)$

Twisted KK-theory for finite group doubles

Why should that help us understand Ostrik's classification of module categories?

- The corresponding modular invariant partition function $\mathcal{Z}$ is a matrix, with rows and columns parametrised by simple objects in the MTC $\mathcal{D}_\omega(G)$.
- In other words, $\mathcal{Z}$ is a linear endomorphism on its fusion ring.
- But that fusion ring is a K-group.
- So $\mathcal{Z}$ is an element in $KK_0(G//_\omega G, G//_\omega G)$
- Up to now, this is just an empty reformulation of what $\mathcal{Z}$ is. Our hope was that maybe it is a special element in that KK-group, that by making explicit this underlying structure, we can see better how $\mathcal{Z}$ depends on (H, ψ).

Correspondences

There is a simple way to describe elements in KK: correspondences (Connes–Skandalis; Emerson–Meyer):

$$X \xleftarrow{f} (Z, E) \xrightarrow{g} Y$$

(unfortunately i can't get latex to work here so i'll use whiteboard....) Multiplication of correspondences is done using pullback.
Let's describe the idea with examples...

Correspondences for finite group doubles

Our work on finite group doubles begins with this observation: to Ostrik's pair (H, ψ), where $H \leq G \times G$ and $\psi \in Z^2(H; \mathbb{T})$, associate the correspondence

$$G//G \xleftarrow{\pi_L} (H//H, \beta) \xrightarrow{\pi_R} G//G$$

where for each $g \in G$, $\beta(g) \in \widehat{C_G(g)}$ is built from ψ.

- The point is that the correspondence does not only describe the matrix, but it defines a module category.

Correspondences for finite group doubles

Our work on finite group doubles begins with this observation: to Ostrik's pair (H, ψ), where $H \leq G \times G$ and $\psi \in Z^2(H; \mathbb{T})$, associate the correspondence

$$G//G \xleftarrow{\pi_L} (H//H, \beta) \xrightarrow{\pi_R} G//G$$

where for each $g \in G$, $\beta(g) \in \widehat{C_G(g)}$ is built from ψ.

- The point is that the correspondence does not only describe the matrix, but it defines a module category.
- We show each of these module categories is irreducible.

Correspondences for finite group doubles

Our work on finite group doubles begins with this observation: to Ostrik's pair (H, ψ), where $H \leq G \times G$ and $\psi \in Z^2(H; \mathbb{T})$, associate the correspondence

$$G//G \xleftarrow{\pi_L} (H//H, \beta) \xrightarrow{\pi_R} G//G$$

where for each $g \in G$, $\beta(g) \in \widehat{C_G(g)}$ is built from ψ.

- The point is that the correspondence does not only describe the matrix, but it defines a module category.
- We show each of these module categories is irreducible.
- We show these module categories are inequivalent except when their pairs (H, ψ) are 'conjugate'.

Correspondences for finite group doubles

Our work on finite group doubles begins with this observation: to Ostrik's pair (H, ψ), where $H \leq G \times G$ and $\psi \in Z^2(H; \mathbb{T})$, associate the correspondence

$$G//G \overset{\pi_L}{\longleftarrow} (H//H, \beta) \overset{\pi_R}{\longrightarrow} G//G$$

where for each $g \in G$, $\beta(g) \in \widehat{C_G(g)}$ is built from ψ.

- The point is that the correspondence does not only describe the matrix, but it defines a module category.
- We show each of these module categories is irreducible.
- We show these module categories are inequivalent except when their pairs (H, ψ) are 'conjugate'.
- Together with Ostrik's classification, this gives a new very explicit description of all module categories for $\mathcal{D}_\omega(G)$.

Extensions of holomorphic orbifolds

Not only are the modular invariants $\mathcal{Z}$ easy to read off, but so are the chiral extensions and isomorphisms of their MTC. Höhn and Huang–Kirillov–Lepowsky explained how chiral extensions is categorical, i.e. that it only depends on the category of representations.

Theorem (Evans–G)

- The set of all extensions of a holomorphic orbifold $\mathcal{A}^G$ correspond to $H = \Delta_K(1 \times N)$ for any subgroup $K \leq G$, where N is normal in K, and any $\psi \in Z^2(H; \mathbb{T})$ with $\psi(k, k; H) = 1$ and $\psi(n, 1; 1, n') = \psi(1, n'; n, 1)$.

The analogue for VOAs will be true once we know holomorphic orbifolds are rational.

Extensions of holomorphic orbifolds

Not only are the modular invariants $\mathcal{Z}$ easy to read off, but so are the chiral extensions and isomorphisms of their MTC. Höhn and Huang–Kirillov–Lepowsky explained how chiral extensions is categorical, i.e. that it only depends on the category of representations.

Theorem (Evans–G)

- The set of all extensions of a holomorphic orbifold $\mathcal{A}^G$ correspond to $H = \Delta_K(1 \times N)$ for any subgroup $K \leq G$, where N is normal in K, and any $\psi \in Z^2(H; \mathbb{T})$ with $\psi(k, k; H) = 1$ and $\psi(n, 1; 1, n') = \psi(1, n'; n, 1)$.
- Branching rules are known.

The analogue for VOAs will be true once we know holomorphic orbifolds are rational.

Extensions of holomorphic orbifolds

Not only are the modular invariants $\mathcal{Z}$ easy to read off, but so are the chiral extensions and isomorphisms of their MTC. Höhn and Huang–Kirillov–Lepowsky explained how chiral extensions is categorical, i.e. that it only depends on the category of representations.

Theorem (Evans–G)

- The set of all extensions of a holomorphic orbifold $\mathcal{A}^G$ correspond to $H = \Delta_K(1 \times N)$ for any subgroup $K \leq G$, where N is normal in K, and any $\psi \in Z^2(H; \mathbb{T})$ with $\psi(k, k; H) = 1$ and $\psi(n, 1; 1, n') = \psi(1, n'; n, 1)$.
- Branching rules are known.
- Corresponding MTC is some explicit twisted double of K/N.

The analogue for VOAs will be true once we know holomorphic orbifolds are rational.

Jones' PhD thesis

Hidden in the proof of this theorem is a result from the PhD thesis of Vaughan Jones. He determined the complete invariants for finite group G actions (not necessarily outer) on the hyperfinite II_1 factor.

- If subgroup N acts by inner automorphisms, then G/N has outer action;

Jones' PhD thesis

Hidden in the proof of this theorem is a result from the PhD thesis of Vaughan Jones. He determined the complete invariants for finite group G actions (not necessarily outer) on the hyperfinite II_1 factor.

- If subgroup N acts by inner automorphisms, then G/N has outer action;
- it has 3-cocycle invariant;

Jones' PhD thesis

Hidden in the proof of this theorem is a result from the PhD thesis of Vaughan Jones. He determined the complete invariants for finite group G actions (not necessarily outer) on the hyperfinite II_1 factor.

- If subgroup N acts by inner automorphisms, then G/N has outer action;
- it has 3-cocycle invariant;
- Jones determines that 3-cocycle.

Jones' PhD thesis

Hidden in the proof of this theorem is a result from the PhD thesis of Vaughan Jones. He determined the complete invariants for finite group G actions (not necessarily outer) on the hyperfinite II_1 factor.

- If subgroup N acts by inner automorphisms, then G/N has outer action;
- it has 3-cocycle invariant;
- Jones determines that 3-cocycle.
- Jones' 3-cocycles exhaust all 3-cocycles on G/N.

Proof of reconstruction

Curiously, our pure extension type module categories have exactly same combinatorial data as Jones did.
The MTC for our chiral extensions are twisted doubles of some G/N, where twist is from original ω multiplied by Jones' 3-cocycle.
We move through different 3-cocycles by varying the 'discrete torsion' ψ.

- We get a conformal net with category of representations $\mathcal{D}_\omega(G)$, by first considering a sufficiently large extension $\tilde{G}$ (as determined by Jones).

Proof of reconstruction

Curiously, our pure extension type module categories have exactly same combinatorial data as Jones did.
The MTC for our chiral extensions are twisted doubles of some G/N, where twist is from original ω multiplied by Jones' 3-cocycle.
We move through different 3-cocycles by varying the 'discrete torsion' ψ.

- We get a conformal net with category of representations $\mathcal{D}_\omega(G)$, by first considering a sufficiently large extension $\tilde{G}$ (as determined by Jones).
- Embed $\tilde{G}$ in some symmetric group S_n.

Proof of reconstruction

Curiously, our pure extension type module categories have exactly same combinatorial data as Jones did.
The MTC for our chiral extensions are twisted doubles of some G/N, where twist is from original ω multiplied by Jones' 3-cocycle.
We move through different 3-cocycles by varying the 'discrete torsion' ψ.

- We get a conformal net with category of representations $\mathcal{D}_\omega(G)$, by first considering a sufficiently large extension $\tilde{G}$ (as determined by Jones).
- Embed $\tilde{G}$ in some symmetric group S_n.
- Take any holomorphic conformal net $\mathcal{A}$; take n copies $\mathcal{A} \otimes \cdots \otimes \mathcal{A}$, then orbifold by permutation action of $\tilde{G}$;

Proof of reconstruction

Curiously, our pure extension type module categories have exactly same combinatorial data as Jones did.
The MTC for our chiral extensions are twisted doubles of some G/N, where twist is from original ω multiplied by Jones' 3-cocycle. We move through different 3-cocycles by varying the 'discrete torsion' ψ.

- We get a conformal net with category of representations $\mathcal{D}_\omega(G)$, by first considering a sufficiently large extension $\tilde{G}$ (as determined by Jones).
- Embed $\tilde{G}$ in some symmetric group S_n.
- Take any holomorphic conformal net $\mathcal{A}$; take n copies $\mathcal{A} \otimes \cdots \otimes \mathcal{A}$, then orbifold by permutation action of $\tilde{G}$;
- Result will be rational net, with MTC some $\mathcal{D}_{\tilde{\omega}}(\tilde{G})$ (in fact $\mathcal{D}_1(\tilde{G})$;

Proof of reconstruction

Curiously, our pure extension type module categories have exactly same combinatorial data as Jones did.
The MTC for our chiral extensions are twisted doubles of some G/N, where twist is from original ω multiplied by Jones' 3-cocycle.
We move through different 3-cocycles by varying the 'discrete torsion' ψ.

- We get a conformal net with category of representations $\mathcal{D}_\omega(G)$, by first considering a sufficiently large extension $\tilde{G}$ (as determined by Jones).
- Embed $\tilde{G}$ in some symmetric group S_n.
- Take any holomorphic conformal net $\mathcal{A}$; take n copies $\mathcal{A} \otimes \cdots \otimes \mathcal{A}$, then orbifold by permutation action of $\tilde{G}$;
- Result will be rational net, with MTC some $\mathcal{D}_{\tilde{\omega}}(\tilde{G})$ (in fact $\mathcal{D}_1(\tilde{G})$;
- thanks to our theorem and Jones' result, some extension of that orbifold will have MTC $\mathcal{D}_\omega(G)$.