Applications of the Generalised Pauli Group in Quantum Information.

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Abstract:

It is known that finite fields with N elements exist only when N is a prime or a prime power. When the dimension N of a finite dimensional Hilbert space is a prime power, we can associate to each basis state of the Hilbert space an element of a finite or Galois field, and construct a finite group of unitary transformations, the generalised Pauli group or discrete Heisenberg-Weyl group [1]. Its elements can be expressed, in terms of the elements of a Galois field through the following equation [2]:

$$V_i^j = \sum_{k=0}^{N-1} \gamma_G^{((k \oplus_G i) \odot_G j)} |k \oplus_G i\rangle\langle k|, \qquad (1)$$

where $\oplus_G$ and $\odot_G$ represent the field operations (addition and multiplication).

When the dimension is a prime p, these operations reduce to the operations modulo p. This group presents numerous applications in Quantum Information Science e.g. tomography, dense coding, teleportation, error correction and so on. For instance the bases that diagonalize the generalised Pauli operators also generalize the X, Y, and Z qubit bases in the sense that they are mutually unbiased or maximally conjugate [3]. Beside, the N^2 elements of the group are in one to one correspondence with generalised Bell states [2] that play a crucial role in many applications of quantum information. The aim of our talk is to give a general survey of these properties and to present recently obtained results in connection with three problems:

-the so-called "Mean King's problem" in prime power dimension, of which the solutions allow us to predict with certainty the value of $N+1$ maximally complementary (conjugate) observables. Such solutions make use of the resources provided by quantum entanglement and covariance properties of Bell states under the Clifford group in order to avoid limitations imposed by the uncertainty principle.

-discrete Wigner distributions [4] that allow us to represent in a N times N discrete phase space any qu$N$it density matrix [5],

-and finally quantum tomography à la von Neumann that is realized by performing measurements of observables that are diagonal in maximally conjugate bases. By doing so, it is possible to minimize redundancies during the acquisition of experimental data that are necessary for the estimation of the coefficients of the density matrix [6, 3] .

References

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