Summer School: Microlocal and Geometric Methods in Representation Theory

Schloss Reisensburg, July 17 –28, 2006

July 13, 2006

1 Preliminary Program

Courses

B. Keller: *Cluster algebras and quiver representations*

T. Kobayashi: *Unitary representations, restrictions, and their applications*

M. Zworski: *Quantum monodromy operator and its applications*

Complementary Lectures

The "Complementary Lectures" will be given by the staff of the Research Training Group Metz-Paderborn. Since they in part are meant to help the participants to follow the three courses, the topics of these lectures in general have not been fixed so far. Here is one example:

J. Hansen: *t.b.a.*

This lecture will contain some background material relevant in Zworski's course.

J. Hilgert: *Characterizing Smooth and Analytic Functions on Compact Lie Groups*

This lecture will contain some background material used in Kobayashi's course.

A. Pasquale *The Plancherel Formula for Real Hyperboloids*

T. Wurzbacher *Normal Forms and Moment Maps in Symplectic and Kähler Geometry*

Research Lectures

The "Research Lectures" will be given by participants who want to present some of the research. Here are the topics announced so far:

P. Clare: *Hilbert C*-modules and intertwining operators*

T. Hartnick: *Flag varieties of infinite-dimensional groups - an axiomatic approach*

G. Morando: *Tempered holomorphic solutions of D-modules on complex curves*

P. Schützdeller: *Convexity properties of moment maps of real forms acting on Kählerian manifolds*

G. Williamson: *Kazhdan-Lusztig Polynomials and Soergel's Bimodules*

S. Wolf: *The composition monoid and the composition algebra of a cyclic quiver.*

Schedule

First week

	Mon	Tue	Wed	Thu	Fri
9:15 - 10:45	Kobayashi	Kobayashi	C (Pasquale)	Kobayashi	Kobayashi
11:15 - 12:45	Zworski	Zworski	Zworski	C	Zworski
15:00 - 16:00	Kobayashi	Kobayashi	C	Kobayashi	Kobayashi
16:15 - 17:15	C (Hilgert)	C (Hansen)	C (Wurzbacher)	C (Wurzbacher)	C
17:30 - 18:30	R	R	R	R	R

Second week

	Mon	Tue	Wed	Thu	Fri
9:15 - 10:45	Keller	Keller	Keller	Keller	Keller
11:15 - 12:45	Zworski	C	Zworski	C	Zworski
15:00 - 16:00	Keller	Keller	Keller	Keller	
16:15 - 17:15	C	C	C (Bierstedt)	C	
17:30 - 18:30	R	R	R	R	

Here "C" means "Complementary Lecture" and "R" means "Research Lecture". The program will will be finalized during the first week.

2 List of Participants (including lecturers)

1) Alldridge, Alexander (Paderborn; alldridg@math.uni-paderborn.de)

2) Bäcklund, Pierre (Uppsala; pierre@math.uu.se)

3) Balleier, Carsten (Paderborn; balleier@math.upb.de)

4) Benameur, Moulay-Tahar (Metz; benameur@math.univ-metz.fr)

5) Bierstedt, Klaus Dieter (Paderborn; klausd@uni-paderborn.de)

6) Bohr, Mathieu (Metz, bohr@univ-metz.fr)

7) Clare, Pierre (Cachan, pierre.clare@gmail.com)

8) Dichev, Nikolay (Paderborn, dichev@math.upb.de)

9) Dicu, Camelia (Cluj-Napoca, cdicu@math.ubbcluj.ro)

10) Dostert, Mike (Metz, dostert@math.univ-metz.fr)

11) Fuchssteiner, Martin (Darmstadt, fuchssteiner@mathematik.tu-darmstadt.de)

12) Graeff, Thomas (Magdeburg, graeff@mathematik.uni-marburg.de)

13) Greb, Daniel (Bochum, daniel.greb@ruhr-uni-bochum.de)

14) Hansen, Sönke (Paderborn, soenke@math.upb.de)

15) Hartnick, Tobias (Darmstadt, tobias.hartnick@gmx.de)

16) Hilgert, Joachim (Paderborn, hilgert@math.uni-paderborn.de)

17) Hitzelsberger, Petra (Darmstadt, hitzelberger@mathematik.tu-darmstadt.de)

18) Johansen, Troels (Paderborn, johansen@math.upb.de)

19) Keliny, Sameh (Paderborn, sameh@math.uni-paderborn.de)

20) Keller, Bernhard (Paris, keller@math.jussieu.fr)

21) Kobayashi, Toshiyuki (Kyoto, toshi@kurims.kyoto-u.ac.jp)

22) Krause, Henning (Paderborn, hkrause@math.uni-paderborn.de)

23) Kurtdere, Ayse (Bochum, kurtdere@gmx.de)

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29) Pasquale, Angela (Metz, pasquale@math.univ-metz.fr)

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32) Rilke, Florian (Paderborn, rilke@math.upb.de)

33) Schnürer, Olaf (Freiburg, schnuerer@pcpool00.mathematik.uni-freiburg.de)

34) Schützdeller, Patrick (Bochum, patrick@cplx.rub.de)

35) Stötzel, Hendrick (Bochum, henrik.stoetzel@rub.de)

36) van der Noort, Vincent (Utrecht, noort@math.uu.nl)

37) Wagner, Stefan (Darmstadt, Leonard-euler@gmx.de)

38) Webster, Benjamin (Berkeley, bwebste@math.berkeley.edu)

39) Wegener Sven-Ake (Paderborn, wegner@math.upb.de)

40) Williamson, Geordie (Freiburg, geordie.williamson@math.uni-freiburg.de)

41) Wolf, Stefan (Paderborn, swolf@math.upb.de)

42) Wolf, Elke (Paderborn, lichte@math.uni-paderborn.de)

43) Wurzbacher, Tilmann (Metz, wurzbacher@math.univ-metz.fr)

44) Zworski, Maciej (Berkeley, zworski@Math.Berkeley.EDU)

3 Abstracts for Research Talks

T. Hartnick: *Flag varieties of infinite-dimensional groups - an axiomatic approach*

The interplay between groups and geometries is a classical topic ever since the work of Felix Klein. It is a remarkable result by Tits that those geometries corresponding to groups of Lie type can be constructed and characterized in a uniform way - they are so-called spherical buildings. This classical result has many generalizstions: E.g. the spherical buildings associated with Lie groups can be characterized by additional topological data. Another recent generalization is the correspondence between so-called twin buildings and certain infinite-dimensional groups called "groups of Kac-Moody type". Combining these two approaches one may ask for a theory of "topological twin buildings", which should then coresond to certain topological groups of Kac-Moody type. The purpose of this talk is to survey some of the classical results, to propose a notion of topological twin buildings and to sketch a structure theory of these buildings and their associated groups.

G. Morando: *Tempered holomorphic solutions of D-modules on complex curves*

First I will introduce my research interests which concern sheaves on sites and D-modules. The former topic has been recently widely developed by M. Kashiwara and P. Schapira (Asterisque 271-284), they realized tempered distributions, tempered holomorphic functions and Whitney C-infinity functions as sheaves on the subanalytic site. Such a work led to the use of such functional spaces with growth conditions in the functorial study of partial differential equations i.e. the study of D-modules. In the second part of the talk I will introduce some recent results I obtained studying tempered holomorphic solutions of D-modules on complex curves.

P. Schützdeller: *Convexity properties of moment maps of real forms acting on Kählerian manifolds*

Let U be a compact connected Lie group and let $U^{\mathbb{C}}$ be the complexification of U. Further let X be a compact connected Kähler manifold with a holomorphic $U^{\mathbb{C}}$-action such that the induced U-action is Hamiltonian, i. e the U-action fixes the symplectic form ω and there exists a moment map $\mu : X \to \mathfrak{u}^*$. By a classical convexity result of Kirwan the intersection of the image of μ with a positive Weyl chamber $\mathfrak{t}_+^*$ in a maximal torus $\mathfrak{t}$ of $\mathfrak{u}$ is a convex polytope.

We discuss a generalization of this statement to the following situation. Let G be a real closed subgroup of $U^{\mathbb{C}}$, which is compatible with the Cartan decomposition $U^{\mathbb{C}} = U \cdot \exp(i\mathfrak{u})$ of $U^{\mathbb{C}}$, i. e. $G = K \cdot \exp(\mathfrak{p})$ where K is a subgroup of U and $\mathfrak{p}$ is a K-stable subspace of $i\mathfrak{u}$. Given such a subgroup G we get a K-equivariant map $\mu_{i\mathfrak{p}} : X \to i\mathfrak{p}^*$ defined by $\mu_{i\mathfrak{p}}(x) = \mu(x)|_{i\mathfrak{p}^*}$ which is the right analog to the moment map μ for the action of G. Let $\mathfrak{a} \subset \mathfrak{p}$ be a maximal subalgebra which is automatically abelian and let $\mathfrak{a}_+$ be the positive Weyl chamber in $\mathfrak{a}$. Then the set $i\mathfrak{a}_+^*$ is a slice for the K-action on $i\mathfrak{p}^*$. We are interested in convexity properties of the set $\mu_{i\mathfrak{p}}(Y) \cap i\mathfrak{a}_+^*$ for closed G-stable subsets Y of X. In particular, we get a convexity result in the following situation.

Let V be a finite dimensional complex $U^{\mathbb{C}}$-representation space. Fixing a U-invariant Hermitian inner product on V we get a moment map $\mu_{\mathbb{P}(V)} : \mathbb{P}(V) \to \mathfrak{u}^*$ for the induced U-action on the projective space $\mathbb{P}(V)$ and the following holds.

Theorem. *Let Y be a closed connected G-stable irreducible semialgebraic subset of $\mathbb{P}(V)$. Then the set $\mu_{i\mathfrak{p}}(Y)$ intersects the positive Weyl chamber $i\mathfrak{a}_+^*$ in a convex polytope.*

S. Wolf: *The composition monoid and the composition algebra of a cyclic quiver.*

Reineke first introduced the composition monoid as a tool for understanding extensions between representations of the Dynkin quivers. He later extended the definition to include all quivers by considering extensions between families of representations. Here I consider the special case of the nilpotent representations of a cyclic quiver and exhibit an isomorphism between the composition monoid and the specialization of the generic composition algebra at $q = 0$.

4 Additional Information

- Reisensburg runs an "Internet-Café". There, it is also possible to print files. Moreover, internet LAN-connections are available in each room (bring your laptop). The public part of Schloss Reisensburg is covered by WLAN, which also extends to part, but not all, of the rooms.
- On Saturday, July 22, we will organize an excursion. On Sunday, July 23, no official activity is planned.
- The email address of the local manager, Mrs. Claudia Ulm, at Reisensburg is *claudia.ulm@uni-ulm.de*. Her phone number is ++49-731-50-38000.

- When you arrive at the train station of Günzburg, you may have to take a taxi to Reisensburg (there are no buses on Sunday). In particular, if you travel in a larger group you may want to order a van. Here is the phone number of the taxi company suggested by Mrs. Ulm: ++49-8221-5584.
- There are a grand piano and several music rooms in Schloss Reisensburg - bring your instrument and sheet music.