Lie Group and Representation Theory Seminar – mini course

Place: Room 402, RIMS, Kyoto University

"Applications of representation theory to problems in analytic number theory". A minicourse.

Joseph Bernstein - Tel Aviv University

In this minicourse I will describe a general approach which allows to use methods of analytic representation theory in order to prove some highly non-trivial estimates in analytic number theory.

This minicourse is based on my works with Andre Reznikov.

I will study representations of the group $G = SL(2, \mathbf{R})$ (and closely related groups) in the space of functions on the automorphic space $X = \Gamma \backslash G$.

My aim is to describe relations of this problem to analytic number theory and to show how using methods of representation theory one can get very powerful estimates of different quantities important in number theory.

The plan of the minicourse.

Lecture 1. Automorphic forms on the upper half-pane.

Abstract. In this lecture I will discuss automorphic forms and Maass forms on upper half-plane.

I will show that many problems about such forms are better expressed in the language of automorphic representations.

I will illustrate this on the model example which gives bounds for Sobolev norms of the automorphic functional.

Lecture 2. Triple product problem. Convexity bound.

Abstract. I will discuss the problem of estimates for triple product of automorphic functions and its connection to estimates of automorphic L-functions.

Using the language of automorphic representations described in first lecture I will show how to explain the exponentially decaying factor in the triple product and then I will describe how to prove the convexity bound for these products.

Lecture 3. Subconvexity bound for triple products.

I will continue the investigation of triple products and show how one can prove a non-trivial subconvexity bound for triple products using a combination of geometric and spectral estimates.

Organizer: T. Kobayashi http://www.kurims.kyoto-u.ac.jp/~toshi/