Lie Group and Representation Theory Seminar, Kyoto 2006

Date: July 25 (Tue), 2006, 16:30–17:30
Place: RIMS, Kyoto University : Room 402
Speaker: Gestur Ólafsson (Louisiana State University)
Title: The Image of the Segal-Bargmann transform Symmetric Spaces and generalizations

Abstract: Let $\Delta = \sum \partial^2/\partial x_i^2$ be the Laplace operator on $\mathbb{R}^n$. The *heat equation* is

$$\begin{aligned} \Delta u(x,t) &= \frac{\partial}{\partial t} u(x,t) \\ \lim_{t\to 0^+} u(x,t) &= f(x) \end{aligned}$$

where f is a L^2-function or a distribution. The solution $u(x,t) = e^{t\Delta} f(x) = H_t f(x)$ is given by

$$H_t f(x) = \int_{\mathbb{R}^n} f(y) h_t(x-y) = \frac{1}{(4\pi t)^{n/2}} \int_{\mathbb{R}^n} f(y) e^{-(x-y)\cdot(x-y)/4t}\, dy$$

where $h_t(x) = 1/(4\pi t)^{n/2} e^{-x\cdot x/4t}$ is the heat kernel, i.e. the solution corresponding to $f = \delta_0$. It can be read of from this explicit formula, that $\mathbb{R}^n \ni x \mapsto H_t f(x)$ has a holomorphic extension to all of $\mathbb{C}^n$. The transform $f \mapsto H_t f \in \mathcal{O}(\mathbb{C}^n)$ is the *Segal-Bargmann transform.* Its image is the space of holomorphic functions $F : \mathbb{C}^n \to \mathbb{C}$, such that

$$\|F\|_t^2 := (2\pi t)^{-n/2} \int |F(x+iy)|^2 e^{-\|y\|^2/2t}\, dxdy < \infty$$

and $\|f\| = \|H_t f\|$.

The Heat equation has a natural generalization to all Riemannian manifolds. The solution is again given by the Heat transform

$$u(x,t) = H_t f(x) = \int f(y) h_t(y)\, dy$$

where h_t is the *heat kernel*, but as there is no *natural* complexification in general it is not clear how to realize the image in a space of holomorphic functions. An exception is the class of symmetric spaces on noncompact type.

In this talk, we start by a short discussion of the Heat transform on $\mathbb{R}^n$ to motivate the main part of the talk and introduce the concepts and ideas that are needed for the Riemannian symmetric spaces of the form G/K where G is a connected noncompact semisimple Lie group and K a maximal compact subgroup. We introduce the natural G-invariant complexification of G/K, called the crown, and describe the image of the Segal-Bargmann transform as a Hilbert space of holomorphic functions on the crown. If time allows, then we will give a different realization of the image space of $L^2(G/K)^K$. That results has a natural formulation for the Heckmann-Opdam setting for positive multiplicity functions. The main tools here are the spherical Fourier transform and the Abel transform.

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