

Lie Group and Representation Theory Seminar

Date: May 15 (Mon), 2006, 16:30–17:30, 17:45–18:45

Place: RIMS, Kyoto University : Room 202

Speaker: Anthony Dooley (University of New South Wales)

16:30–17:30

Title: Intertwining operators, the Cayley transform, and the contraction of K to NM

Abstract: If $G = KAN$ is the Iwasawa decomposition of a rank one semi-simple Lie group, it is interesting to use harmonic analysis on N together with the N picture of the principal and exceptional series to analyse the representation theory. In particular, the author recently proved a representation-theoretic version of the Cowling-Haagerup theorem on the approach to the identity by uniformly bounded representations. In order to establish the Baum-Connes conjecture “with coefficients”, one needs information about the K picture, and it turns out that this can be obtained from this result together with the study of the contraction of K to NM .

17:45–18:45

Title: Orbital convolution theory for semi-direct products

Abstract: Dooley and Wildberger in the setting of compact groups, introduced the wrapping map Φ . This map associates, to each Ad-invariant distribution μ of compact support on the Lie algebra \mathfrak{g} , a central distribution $\Phi\mu$ on the Lie group G , via the formula, for $f \in C_c^\infty(G)$,

$$\langle \Phi\mu, f \rangle = \langle \mu, j \cdot f \circ \exp \rangle, \quad (1)$$

where j is the square root of the Jacobian of $\exp : \mathfrak{g} \rightarrow G$.

Φ provides a convolution homomorphism between the Euclidean convolution structure on \mathfrak{g} and the group convolution on G , that is

$$\Phi(\mu *_{\mathfrak{g}} \nu) = \Phi\mu *_{G} \Phi\nu. \quad (2)$$

This mapping is a global version of the Duflo isomorphism — there are no conditions on the supports of μ and ν (they need not, for example, lie in a fundamental domain). We may interpret the dual of Φ , a map from the Gelfand space of $M_G(G)$ to that of $M_G(\mathfrak{g})$, in such a way as to obtain the Kirillov character formula for G .

In a recent paper Andler, Sahi and Torrosian have extended the Duflo isomorphism to arbitrary Lie groups. Their results give a version of equation which holds for germs of hyperfunctions with support at the identity.

Our result can be viewed as a statement that, for compact Lie groups, the results of hold for invariant distributions of compact support, and hold globally in the sense that the restriction that the supports are compact is needed only in order to ensure that the vconvolutions exist. This observation allows one to develop calculational tools for invariant harmonic analysis based on convolutions of orbits and distributions in the Euclidean space \mathfrak{g} .

We have extended these ideas to semi-direct product groups $G = V \rtimes K$, where V is a vector space and K a compact group. There are several significant differences between this case and the compact case — firstly, there is no identification between the adjoint and coadjoint pictures as the Killing form is indefinite, and secondly, perhaps more significantly, the fact that the orbits are no longer compact means that there are few Ad-invariant distributions of compact support — so the convolutions in formula (2) need careful interpretation.

いつもと曜日・場所が違いますのでご注意ください。

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