The seventh Takagi Lectures

November 21, 2009 (Sat) 15:00–16:00
November 22, 2009 (Sun) 16:30–17:30
Graduate School of Mathematical Sciences
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Mori dream spaces

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Abstract

A fundamental result of Hilbert says that if R is a finitely generated $\mathbb{C}$-algebra then the ring of invariants R^G is finitely generated, provided G is a reductive algebraic group (for example products of the multiplicative group $(\mathbb{C}^*)^k$). Nagata gave examples where R is a polynomial ring and $G = \mathbb{C}^k$ is a product of the additive group and yet R^G is not finitely generated. In fact R^G is the total coordinate ring of a blow up of projective space $\mathbb{P}^n$.

If X is a projective variety and the Cox ring is finitely generated then X is called a Mori dream space; every toric variety and every Fano variety is a Mori dream space. As the name might suggest, Mori dream spaces have very many nice properties; every section ring is finitely generated; flips always terminate; there is a natural combinatorial structure to the set of minimal models, and all of this controlled by the geometric invariant theory of some Thaddeus master space. We will explore this circle of ideas in the talk.