The 13th Takagi Lectures

November 16 (Sat)–17 (Sun), 2013
Research Institute for Mathematical Sciences
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ABSTRACT

H. Oh:
Apollonian Circle Packings: Dynamics and Number Theory

An Apollonian circle packing is an ancient Greek construction which is made by repeatedly inscribing circles into the triangular interstices of four mutually tangent circles, via an old theorem of Apollonius of Perga (262–190 BC). They give rise to one of first examples of a fractal in the plane. In the first lecture, we will discuss recent results on counting and distribution of circles in Apollonian packings in fractal geometric terms and explain how the dynamics of flows on infinite volume hyperbolic manifolds are related.

A beautiful theorem of Descartes observed in 1643 implies that if the initial four circles have integral curvatures, then all the circles in the packing have integral curvatures, as observed by Soddy, a Nobel laureate in Chemistry. This remarkable integrality feature gives rise to several natural Diophantine questions about integral Apollonian packings such as "how many circles have prime curvatures". In the second lecture, we will discuss progress on these questions while introducing recent developments on expanders and the affine sieve by Bourgain–Gamburd–Sarnak.

G. Tian:
Kähler–Einstein Metrics on Fano Manifolds

The study of Kähler–Einstein metrics was initiated by E. Calabi in 50's. In 70s, Yau and Aubin solved the existence problem for Kähler–Einstein metrics on compact Kähler manifolds with vanishing or negative first Chern class. Since then, it has been a challenging problem to studying the existence of Kähler–Einstein metrics on Fano manifolds. A Fano manifold is a compact Kähler manifold with positive first Chern class. There are obstructions to the existence of Kähler–Einstein metrics on Fano manifolds, first by Matsushima in late 50s, secondly by A. Futaki in early 80s and also K-stability in 90s. These lectures will concern Kähler–Einstein metrics and K-stability.

In the first lecture, I will give a brief tour on the study of Kähler–Einstein metrics on Fano manifolds in last two decades and then discuss recent solution for the existence of Kähler–Einstein metrics on Fano manifolds which are K-stable. I will also discuss key tools used in the solution.

In the second lecture, I will focus on the K-stability, its original definition as well as new formulations. I will discuss some recent works on K-stability, particularly, S. Paul's work on stable pairs, which generalize certain fundamental results in the Geometric Invariant Theory, and show how the K-stability can be put in this general setting. Some open problems may be discussed in the end if time permits.