

Non-local Poisson structures and applications to the theory of integrable systems

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Dedicated to Minoru Wakimoto on his 70th birthday

Abstract. We develop a rigorous theory of non-local Poisson structures, built on the notion of a non-local Poisson vertex algebra. As an application, we find conditions that guarantee applicability of the Lenard–Magri scheme of integrability to a pair of compatible non-local Poisson structures. We apply this scheme to several such pairs, proving thereby integrability of various evolution equations, as well as hyperbolic equations.

Keywords and phrases: non-local Poisson vertex algebra, non-local Poisson structure, rational matrix pseudo-differential operators, Lenard–Magri scheme of integrability, bi-Hamiltonian integrable hierarchies

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