

Ricci curvature and measures^{*}

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Abstract. In the last thirty years three *a priori* very different fields of mathematics, optimal transport theory, Riemannian geometry and probability theory, have come together in a remarkable way, leading to a very substantial improvement of our understanding of what may look like a very specific question, namely the analysis of spaces whose Ricci curvature admits a lower bound. The purpose of these lectures is, starting from the classical context, to present the basics of the three fields that lead to an interesting generalisation of the concepts, and to highlight some of the most striking new developments.

Keywords and phrases: Ricci curvature, geometry of spaces of measures, Bakry–Émery estimate, optimal transport theory, lower bounds on curvature, Wasserstein distances, metric measured spaces, Gromov–Hausdorff topology, entropy functionals

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