On some number-theoretic conjectures of V. Arnold

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Abstract. In [1], V.I. Arnold conjectured “the matrix Euler congruence”: \( \text{tr} A^p \equiv \text{tr} A^{p-1} \pmod{p^n} \) for any integer matrix \( A \), prime \( p \), and natural number \( n \). He proved it for \( p \leq 5, n \leq 4 \). In fact the conjecture immediately follows from a result of C.J. Smyth [5]. We give a simple proof of this result and discuss a related conjecture of Arnold concerning some congruences for multinomial coefficients.

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