

Hilbert schemes of lines and conics and automorphism groups of Fano threefolds*

Alexander G. Kuznetsov · Yuri G. Prokhorov · Constantin A. Shramov

Received: 11 April 2017 / Revised: 11 November 2017 / Accepted: 22 November 2017

Published online: 14 February 2018

© The Mathematical Society of Japan and Springer Japan 2018

Communicated by: Takeshi Saito

* The authors were partially supported by the Russian Academic Excellence Project “5-100”, by RFBR grant 15-01-02164, and by the Program of the Presidium of the Russian Academy of Sciences No. 01 “Fundamental Mathematics and its Applications” under grant PRAS-18-01. A.K. was also supported by RFBR 14-01-00416, 15-51-50045 and by the Simons foundation. Yu.P. was also supported by RFBR 15-01-02158 and 15-51-50045. C.S. was also supported by RFBR 14-01-00160 and 15-01-02158 and the Dynasty foundation.

A.G. KUZNETSOV

Steklov Mathematical Institute of Russian Academy of Sciences, 8 Gubkina street, Moscow, 119991, Russia,

The Poncelet Laboratory, Independent University of Moscow, Bolshoy Vlasievskiy Pereulok 11, 119002, Russia and

Laboratory of Algebraic Geometry, National Research University Higher School of Economics, Usacheva street, 6, Moscow, 119048, Russia

(e-mail: akuznet@mi.ras.ru)

YU.G. PROKHOROV

Steklov Mathematical Institute of Russian Academy of Sciences, 8 Gubkina street, Moscow, 119991, Russia,

Laboratory of Algebraic Geometry, National Research University Higher School of Economics, Usacheva street, 6, Moscow, 119048, Russia and

Department of Algebra, Moscow State University, Moscow, 119991, Russia

(e-mail: prokhoro@mi.ras.ru)

C.A. SHRAMOV

Steklov Mathematical Institute of Russian Academy of Sciences, 8 Gubkina street, Moscow, 119991, Russia and

Laboratory of Algebraic Geometry, National Research University Higher School of Economics, Usacheva street, 6, Moscow, 119048, Russia

(e-mail: costya.shramov@gmail.com)

Abstract. We discuss various results on Hilbert schemes of lines and conics and automorphism groups of smooth Fano threefolds of Picard rank 1. Besides a general review of facts well known to experts, the paper contains some new results, for instance, we give a description of the Hilbert scheme of conics on any smooth Fano threefold of index 1 and genus 10. We also show that the action of the automorphism group of a Fano threefold X of index 2 (respectively, 1) on an irreducible component of its Hilbert scheme of lines (respectively, conics) is faithful if the anticanonical class of X is very ample except for some explicit cases.

We use these faithfulness results to prove finiteness of the automorphism groups of most Fano threefolds and classify explicitly all Fano threefolds with infinite automorphism group. We also discuss a derived category point of view on the Hilbert schemes of lines and conics, and use it to identify some of them.

Keywords and phrases: Fano variety, Hilbert scheme, automorphism group, line, conic, derived category

Mathematics Subject Classification (2010): 14J45, 14J50, 14J30, 14C05
