

## Towards a Lie theory of locally convex groups

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**Abstract.** In this survey, we report on the state of the art of some of the fundamental problems in the Lie theory of Lie groups modeled on locally convex spaces, such as integrability of Lie algebras, integrability of Lie subalgebras to Lie subgroups, and integrability of Lie algebra extensions to Lie group extensions. We further describe how regularity or local exponentiality of a Lie group can be used to obtain quite satisfactory answers to some of the fundamental problems. These results are illustrated by specialization to some specific classes of Lie groups, such as direct limit groups, linear Lie groups, groups of smooth maps and groups of diffeomorphisms.

*Keywords and phrases:* infinite-dimensional Lie group, infinite-dimensional Lie algebra, continuous inverse algebra, diffeomorphism group, gauge group, pro-Lie group, BCH–Lie group, exponential function, Maurer–Cartan equation, Lie functor, integrable Lie algebra

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