Analogy between $\ell$-adic sheaves in $\mathbb{Z}_\ell$ and $\mathbb{Q}_\ell$ modules on $\mathcal{X}$ and wild ramification

Chan cycle

Cycles of div on the cotangent b'dle

How to define it?

A possible approach - vanishing cycles

$\mathcal{X}/\mathbb{F}_p$, smooth, closed $\rightarrow \ell$-adic sheaf

div closed $\mathbb{F}_\ell$-sheaf

$f: \mathcal{X} \rightarrow \mathbb{C}$, flat morphism to smooth curve

$\phi^g_{u}(f, f)$

complex of vanishing cycles

$\nu_u(f, t)$

finite action of the absolute Galois group

of $K_v$, $v$ = finite

$\dim \text{tot} \phi^g_{v}(f, t)$ $= 0$ under OSF

local cyclicity of smooth morphism

$\phi_{v}(f, t) = 0$ if $f$ is smooth and $f$ locally constant

Use $\Phi_{v}$ for

vanishing total dim

of $\phi_{v}(f, t)$ to measure sing of $f$

and ramification of $f$

$\Phi_{v}$: Characteristic Cycle Cycle on $\mathcal{T}_x$

Existence of $\text{Sing Supp}$ = $\Phi_{v}$ of Characteristic cycle

Milnor formula: Euler-Poincare formula
Suppose there exists a closed cone subset $S$ of $\text{div}(\text{Sing})$ satisfying

$$S \subset \Gamma^* X = V(\Delta X)$$

and is stable under rescaling $t \mapsto t$

$$(SS1) \quad \text{for } f : C \to \text{famly}$$

- $W \times X \times B$
- $f$ flat, non characteristic w.r.t. $S$
- $W \times T^* \to TW/B \to W \times T^*_B$

$\Rightarrow f$ is (semi)locally acyclic rel. to the pull-back of $\Delta^2$

example of the vanishing of vanishing cycles

$S \text{ SS} = ?$ singular support

Example 1 $X = X - D$, SNC $U = X - D, F$ l.c.c. + family

$\text{Sing} D$: $D = \{0\}$

$SS(d, 7) = U / D \times X$

- $X$ curve $\Rightarrow$ l.c.c. or $U \times X$
- $SS(7) = T^*_X U / T^*_X \times X$
- $D = \{0\}$ fibers

1. 2. Lagaeanum. $\dim X \leq 2$, wildly ame.

Ramification theory $\Rightarrow \exists \Sigma_c X$. $\alpha X - 2$ SS $\Rightarrow$ is defined

$\Rightarrow \dim X \leq 2 = SS$ is defined.
Then suppose SSK exists. There exists a unique C-linear combination \( C \frac{V}{K} \otimes \mathcal{O}(\mathcal{R}) \) of invloved comp of \( \mathcal{S} = \mathcal{U} \mathcal{C} \) with \( \mathcal{R} \in \mathbb{Z} \) such that 

\[
- \text{diag} \circ \phi_{\mathcal{V}}(K, f) = C \left( \text{Ch} \frac{V}{K}, df \right) \quad \forall \mathcal{V}
\]

for every morphism \( \mathcal{V} \to C \) on \( \alpha \text{-an} \). Let \( \mathcal{U} \) be a closed pt such that \( \alpha \mathcal{U} \) is an isolated char point of \( f \). \( \text{w.r.t.} \) the pull-back of SSK

\[
\forall \mathcal{V} \times T^c C \to T^c V \supset \text{SSK}
\]

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\]

\[
\text{Isolated char} = \left\{ \text{SSK} \land \forall \mathcal{V} \text{ for } \mathcal{V} \right\}
\]

\[
\text{V.S. int. multiplicity}
\]

**Definition** Coefficients \( \alpha \):

Assume \( X \) quasi-projective, \( L \) very ample.

\[
\mathcal{X} \times \mathcal{B} \to \mathbb{P}(\mathcal{X} \times \mathcal{B}; T^c \mathcal{P}) 
\]

\[
\mathcal{D} \to \mathcal{D}^v
\]

\[
\text{Univ. fam. of hyperplane sections}
\]

\[
G = \mathcal{G}(1, \mathcal{D}^v) \quad a_\mathcal{G} = (-1)^{d-1}
\]

\[
\text{Aut. conductor of}
\]

\[
\text{Milnor families for generic pencil}
\]

Continuity of the Swan conductor Deligne-Langlands

Milnor families for every unsplit reduction of \( \mathcal{R} \).

More assumption \( \Rightarrow \) Compatibility with generic reduction

\[
\text{Euler-Poincare by induction on } d
\]