X smooth $/k$ $D \subset X$ div. SNC $d = \dim X$

$j : U = X \setminus D \hookrightarrow X$ open imm

$\Lambda \ (= \mathbb{Z}/\ell^n)$ $\mathcal{F}$ U 上の l.c.c. sheaf, flat Λ-module

$\mathcal{F}$ は D にそって tamely ramified

U 上の finite étale scheme で表現される

$C(j_!\mathcal{F}) \in H^{2d}(X, \Lambda(d))$ は $\mathrm{rk}\,\mathcal{F} \cdot (-1)^d C_d(\Omega^1_X(\log D))$

$$\begin{array}{ccccccc} (X \times X)' & \xleftarrow{k_1 \circ k_2} & (X\times X)^{\sim} & = & X \times X & \xleftarrow{\hat{j}} & U \\ \pi \downarrow & & & & & & \\ X \times X & & & & & & \end{array}$$

$\mathcal{H} = R\mathcal{H}om(pr_2^* j_!\mathcal{F}, R\,pr_1^! j_!\mathcal{F})$ $\quad D_{ctf}(X \times X, \Lambda) = D(X \times X)$

$id \in H^0_X(X \times X, \mathcal{H}) \to H^{2d}(X, \Lambda(d))$

$\cup$

$C(j_!\mathcal{F}) = \mathrm{rk}\,\mathcal{F} \cdot (X, X)_{X \times X}$

これを示したい.

$$\mathcal{H}' = k_{1!}\, Rh_{2*}(\hat{j}_* \mathcal{H}om(pr_2^* \mathcal{F}, pr_1^* \mathcal{F}))(d)[2d]$$

$$D((X\times X)')$$

1. coh が変わらない。
2. 分岐が消えている。

$$H^{2d}_x(X\times X, \hat{j}_*\mathcal{H}om(\mathcal{F},-)(d)).$$

$\downarrow$ want: injective

$$\mathrm{id} \longmapsto ?$$

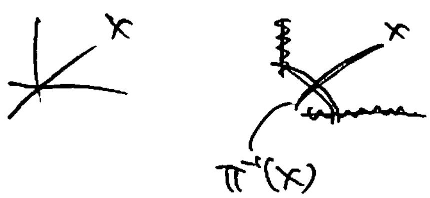
$$H^0_x(X\times X, \mathcal{H}) \xrightarrow{\sim} H^0_{\pi^{-1}(x)}((X\times X)', \mathcal{H}') \hookrightarrow H^0(U\times U, \mathcal{H}|_{U\times U})$$

ここが同形と ここが injective は同値

$$X\times X \longleftarrow (X\times X)' \supset X\times X$$

$$\cup$$

$$X$$



分岐が消えていることの使い方

$R = \sum r_i D_i$ r_i integer ≥ 0 (tame なら $R=0$)

$\mathcal{F}$: U上の l.c.c. Λ-module, log 分岐 $\leq R+$

$$\begin{array}{ccc} U & \xrightarrow{\delta_U} & U\times U \\ j \downarrow & \square & \downarrow \leftarrow j^{(R)} \\ X & \xrightarrow{\delta^{(R)}} & (X\times X)^{(R)} \end{array}$$

$$\delta^{(R)*} \hat{j}^{(R)}_* \longrightarrow j_* \delta_U^*$$

base change

No.

Date

$$\delta^{(R)*} j_*^{(R)} \mathcal{H}om(pr_2^*\mathcal{F}, pr_1^*\mathcal{F}) \to j_* \delta_U^* \underbrace{\mathcal{H}om(pr_2^*\mathcal{F}, pr_1^*\mathcal{F})}_{= \mathcal{E}nd(\mathcal{F})}$$

Lem

$\mathcal{F}$: log 分岐 $\leq R+$ $\Rightarrow$ これは同形

証明

主張は $(X \times X)^{(R)}$ 上 X の近傍で étale local

$$\begin{tikzcd}
 & Z_0 \arrow[r, hook, "open"] \arrow[dr] & Z \arrow[d, "f.étale"] & W = V\times V/\Delta G \arrow[l, hook'] \arrow[d] & V \arrow[d, "\begin{array}{l}\text{fin. étale}\\ G\text{-torsor}\end{array}"] \\
X \arrow[ur] \arrow[rr, "\delta^{(R)}"] & & (X\times X)^{(R)} & \supset U\times U & U
\end{tikzcd}$$

U log. ram. bdd $\leq R$

$\mathcal{H}om|_W = (pr_2^* \mathcal{E}nd(\mathcal{F}))|_W$ を示す

$\mathcal{F} \leftrightarrow G$ の表現 M $\qquad$ $Hom_\Delta(M_2, M_2)$

$Hom_\Delta(M_2, M_1)$ $\quad$ $G \times G$ の表現

$\Delta G \subset G \times G \overset{pr_1}{\underset{pr_2}{\rightrightarrows}} G \curvearrowright M$

ΔG に制限すると, $M_2 = M_1$

$\therefore \mathcal{H}om|_W = (pr_2^* \mathcal{E}nd(\mathcal{F}))|_W$.

No.

Date

$$\begin{tikzcd}
Z_0 \arrow[r, hook, "j_{Z_0}"] \arrow[d] & W \arrow[d] \arrow[dd, bend left=60, "p_2"] \\
(X\times X)^{(P)} \arrow[r, hook] \arrow[d, "\delta'"] & U\times U \arrow[d, "pr_2"] \\
X \arrow[r, hook] & U
\end{tikzcd}$$

étale, smooth, smooth, p_2

$\mathcal{E}nd(\mathcal{F})$

smooth base change thm

$$p_2^* j_* \mathcal{E}nd(\mathcal{F}) \longrightarrow j_{Z_0 *} p_2^* \mathcal{E}nd(\mathcal{F}) \quad \text{は同型}$$

さらに δ'^* をとると

$$\cancel{\delta'^* p_2^*} j_* \mathcal{E}nd(\mathcal{F}) \to \delta'^* j_{Z_0 *} \underbrace{p_2^* \mathcal{E}nd(\mathcal{F})}_{\mathcal{H}om}$$

今考えている射の逆が同型 $\qquad Z_0$- //

$$H^0_X(X\times X, \mathcal{H}) \longrightarrow H^0_{\pi^{-1}(X)}((X\times X)', \mathcal{H}') \longrightarrow H^0(U\times U, \mathcal{H}|_{U\times U})$$

$\mathrm{id} \in H^0_X(X\times X,\mathcal{H})$, $\quad [\mathrm{id}]^\vee[X] \in H^0_{\pi^{-1}(X)}((X\times X)', \mathcal{H}')$

$$\uparrow$$

$$H^{2d}_X(X\times X, \tilde{j}_* \mathcal{H}om(-,-)(d))$$

$\underline{\delta^* \times \delta^! \to \delta^!}$

$$\uparrow$$

$$H^0(X, \tilde{\delta}^* \tilde{j}_* \mathcal{H}om) \times H^{2d}_X(X\times X, \Delta(d))$$

$$\mathrm{id} \in H^0(X, j_* \mathcal{E}nd(\mathcal{F})) \xrightarrow{\ \sim\ } H^0(X, \tilde{\delta}^* \tilde{j}_* \mathcal{H}om), \qquad [X] \in H^{2d}_X(X\times X, \Delta(d))$$

$id \in H^0_{\pi^{-1}(X)}((X\times X)', \mathcal{H}')$ は $[id]^{\cup}[X]$ と等しい。

$$\begin{array}{ccc} & & (\hookrightarrow H^0(U\times U, \mathcal{H}|_{U\times U}) \text{ 単射がわかっているから}) \\ \tilde{\delta}^* \downarrow & & \\ H^0(X, \mathcal{H}'|_X) & \longleftarrow & H^0(X, j_*\mathcal{E}nd(\mathcal{F})) \times H^{2d}(X, \Lambda(d)) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \qquad\qquad \rotatebox{90}{$\in$} \\ e & \longleftarrow\!\shortmid & (\ id \quad , \quad (X,X)_{X\times X}\ ) \end{array}$$

分岐がないときと同様に

$$c(j_!\mathcal{F}) = \mathrm{rk}\,\mathcal{F}\cdot \underline{(X,X)_{X\times X}} \in H^{2d}(X, \Lambda(d))$$

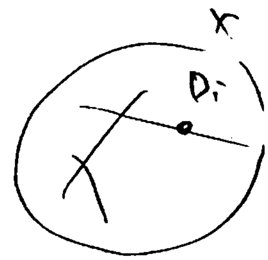
がわかる.

wildなとき $R = \sum r_i D_i,\ r_i > 0$ integer

$\mathcal{F}$: U上 l.c.c. log分岐 $\le R+$
rk 1.

cleanness 条件 $\chi : \pi_1(U)^{ab} \to \Lambda^\times$
(Fに対応する指標)

D_i 既約成分 K_i 局所体



$\chi_i : G_{K_i}^{ab} \to \pi_1(U)^{ab} \xrightarrow{\chi} \Lambda^\times \supset \mu_p$

log分岐 $\le R+ \ \to\ \chi_i|_{G^{r_i+}_{K_i,\log}} = 1$

$\chi_i \in \mathrm{Hom}(\underbrace{\mathrm{Gr}^{r_i} G_{K_i,\log}}_{\text{pで消える}}, \mu_p) \xhookrightarrow{rsw} \mathrm{Hom}_{F_i}(m_{K_i}^{r_i}/m_{K_i}^{r_i+1}, \Omega^1_X(\log D) \otimes F_i$

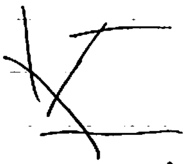
$\chi_i \longmapsto rsw(\chi_i)$

$\otimes_{F_i} \ell$

$\exists!\ \mathrm{rsw}\,\chi \in \Gamma(D, \Omega^1_X(\log D)(R))$ $\quad \Omega^1_X(\log D)(R)_{\xi_i} = \mathcal{H}om($ $)$

s.t. $(\mathrm{rsw}\,\chi)_{\xi_i} = \mathrm{rsw}(\chi_i)$ $\quad \xi_i$: D_i の gen. pt.

$K_i = K(\xi_i)$



$(X * X)^{(R)} \underset{j^{(R)}}{\supset} U \times U$

$j^{(R)}_* \mathcal{H}om(\mathrm{pr}_2^* \mathcal{F}, \mathrm{pr}_1^* \mathcal{F})$ は $(X * X)^{(R)}$ 上の smooth sheaf

$E = (X * X)^{(R)} \setminus (U \times U)$

$\downarrow$ vector bdle

D

$\mathrm{rk}\,\mathcal{F} = 1$ $\quad$ log 分岐 $\leq R+ \Longleftrightarrow \forall i$ で $G_{K_i}^{r_i}$ が自明に作用

(inertia の p-sylow が normal と仮定すれば $\Leftarrow$)

Zariski-Nagata

$\forall i$ ... E の各既約成分の gen. pt で $\mathcal{H}om(-,-)$ が不分岐

$\overset{\text{Zariski-Nagata}}{\Longrightarrow}$ E にそって不分岐 ($\Longleftrightarrow j^{(R)}_* -$ が smooth)

$\Rightarrow \exists\ \mathrm{rsw}\,\chi \in \Gamma(D, -)$

Assume rsw χ がいたるところ non-zero (Cleanness condition

$\Rightarrow c(j_! \mathcal{F}) = (X, X)_{(X*X)^{(R)}}$

$\shortparallel$

$(-1)^d c_d(\Omega^1_X(\log D)(R))$

No.

Date

証明

1. Coh が変わらない.
2. 符号を消した.

$$\begin{array}{c}(X\times X)^{(R)}\\ \downarrow \pi^{(R)}\\ X\times X\end{array}$$

$\mathcal{H}$ on $X\times X$

$$\mathcal{H}^{(R)} = j^{(R)}_{*}\mathcal{H}om(pr_2^{*}\mathcal{F}, pr_1^{*}\mathcal{F})(d)[2d]$$

$$H^0_X(X\times X, \mathcal{H}) \to H^0_{\pi^{(R)-1}(X)}((X\times X)^{(R)}, \mathcal{H}^{(R)})$$

$$id \longmapsto [id]^{\cup}(X)$$

$$H^0_X((X\times X)^{(R)}, \mathcal{H}^{(R)}) \qquad H^0_U(U\times U, \mathcal{H}|_{U\times U}) \ni (\ast)$$

$$\uparrow$$

$$H^0(X, \delta^{(R)*}j^{(R)}_{*}\mathcal{H}om(-\,-)) \times H^{2d}_X((X\times X)^{(R)}, \Lambda(d)) \ni [X]$$

$$② \downarrow \wr$$

$$H^0(X, j_* \mathcal{E}nd(\mathcal{F})) \ni id$$

$$id \longmapsto [id]^{\cup}[X] \Rightarrow C(j_! \mathcal{F}) = (X, X)_{(X\times X)^{(R)}}$$

さっきのとおり.

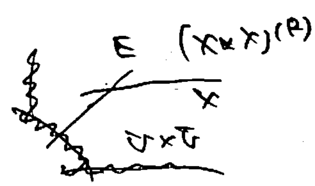
injectivity を示す.

(*)をいいたい.

$$H^0_E((X\times X)^{(R)}, \mathcal{H}^{(R)}) \to H^0_{\pi^{-1}(X)}((X\times X)^{(R)}, \mathcal{H}^{(R)}) \to H^0_U(U\times U, \mathcal{H}|_{U\times U})$$

0をいえばよい.

exact.



$$U\times U = (X\times X)^{(R)} \setminus E$$

No.

Date

$$H^0_E((X*X)^{(R)}, j_*^{(R)}\mathcal{H}om(-,-)(d)[2d])$$

$$= H^0(E, Ri_E^!\mathcal{H}^{(R)})$$

$$= H^0(D, \underbrace{R\pi_{E*}Ri_E^!\mathcal{H}^{(R)}})$$

$$\begin{array}{ccc} E & \xrightarrow{i_E} & (X*X)^{(R)} \\ \pi_E \downarrow & & \downarrow \\ D & \longrightarrow & X \end{array}$$

$\overset{\shortparallel}{0}$ をいえばよい.

dual

$$D_D(R\pi_{E*}Ri_E^!\mathcal{H}^{(R)}) = R\pi_{E!}D_E(Ri_E^!\mathcal{H}^{(R)})$$

$$= R\pi_{E!}i_E^*\underline{D_{(X*X)^{(R)}}\mathcal{H}^{(R)}}$$ 0をいえば

$$\cap$$

$$j_*^{(R)}\mathcal{H}om(pr_1^*\mathcal{F}, pr_2^*\mathcal{F})(-)(-)$$

D の各 geom. pt. $\bar{x}$ に対し,

$$H^q_c(E_{\bar{x}}, \underline{j_*^{(R)}\mathcal{H}om(-,-)|_{E_{\bar{x}}}}) = 0 \quad (\forall q)$$

をいえばよい. (proper base change theorem)

$k(\bar{x})$ 上の vector space　　Artin-Schreier sheaf

$t^p - t = f$

$f_{\bar{x}}$: $E_{\bar{x}}$ 上の linear form

$$E_{\bar{x}} = \mathbb{V}(\Omega^1_X(\log D)(R)) \times \bar{x} = \underline{(\Omega^1_X(\log D)(R) \otimes k(\bar{x}))^*}$$

$\rotatebox{90}{$\in$}$

$f_{\bar{x}} = \mathrm{rsw}\,\chi$ の $\bar{x}$ での germ

$\neq$

0 cleanness

No.

Date

$F = \overline{F}$ char $p > 0$ $\overline{F} = F$

V 有限次 F-vector sp. $f \neq 0$ linear form

$\mathcal{L}$ $t^p - t = f$ で定まる V 上の smooth sheaf rk 1

A-S sheaf

とすると、$H_c^q(V, \mathcal{L}) = 0$ $(\forall q)$

(レポート)