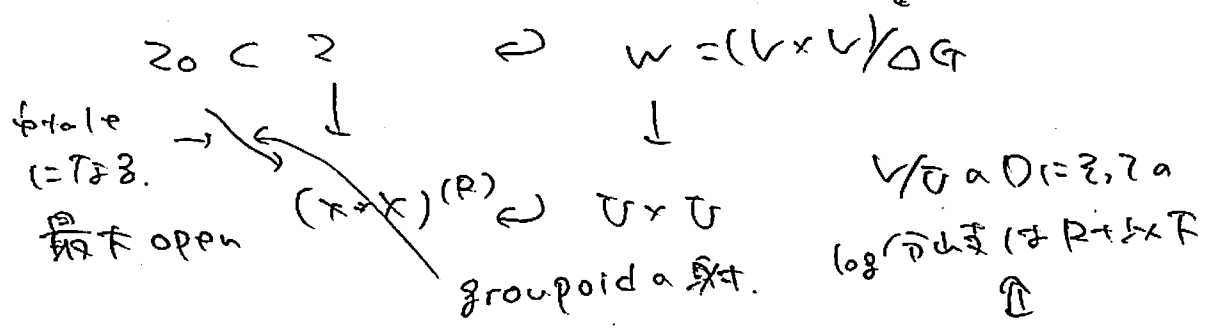


k : 完全体 $ch = p > 0$

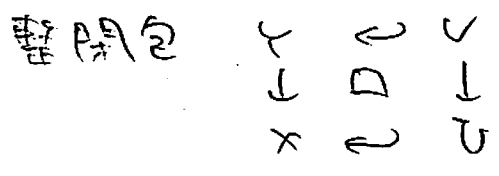
X : smooth $\supset D$ S.N.C.D
 \cup $\cup_i D_i$
 $U = X \setminus D$

$R = \sum r_i D_i \quad (r_i \in \mathbb{N})$

$V \rightarrow U$ G -torsor G 有限群 G 非可換 G 作用自由



log branch 群との関係



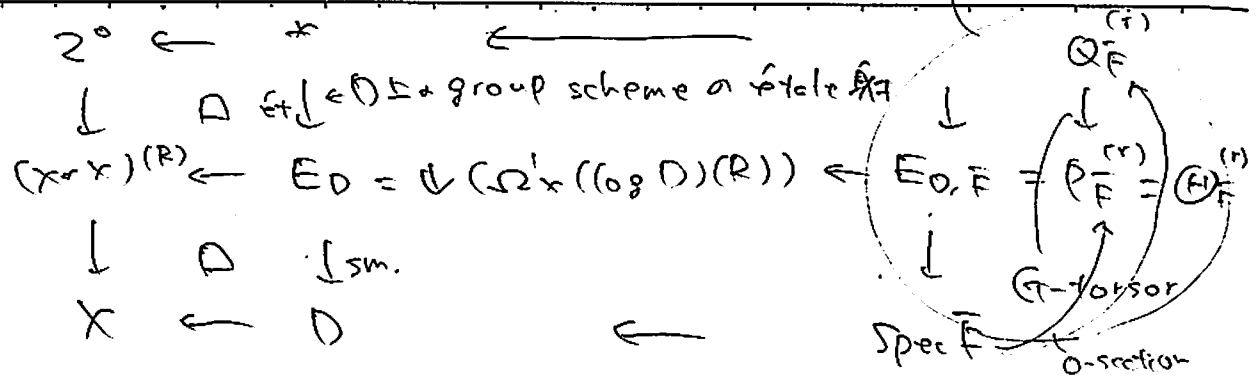
$k = \text{Frac } \hat{\mathcal{O}}_{X, \xi}$

$L = \text{Frac } \hat{\mathcal{O}}_{U, \eta}$

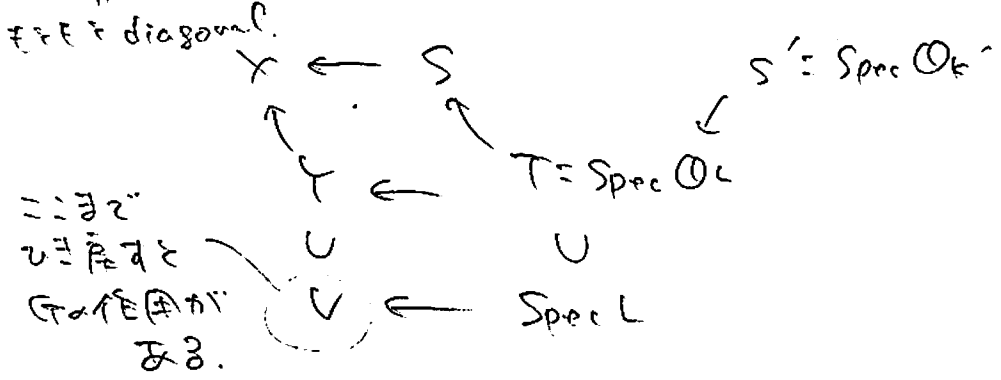
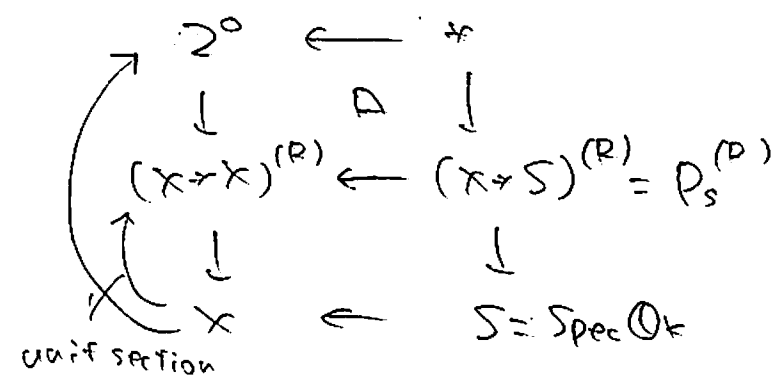
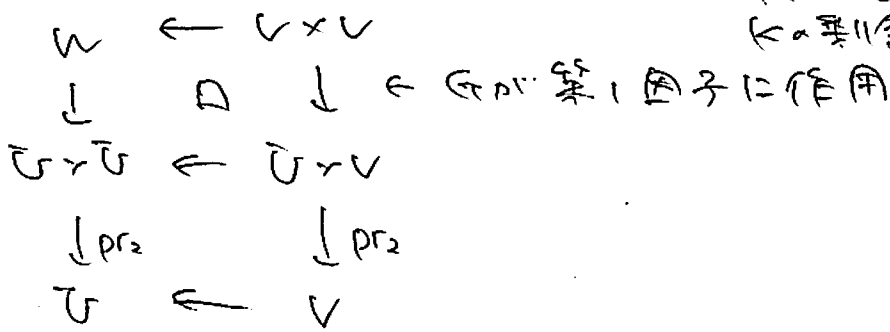
$R = rD \quad G^{rt} = \{1\}$

D : 除数
 Y : smooth
 $E = Y \setminus V$ smooth
 $\xi \in D$ gen. pt
 $\eta \in E$ "
 L is k Galois field
 $\text{Gal}(L/k) = G$

カテゴリー理論
L-圏の有限性



$F = k(\bar{F}) \cap \alpha$ 有限体
 $k \alpha$ 剰余体



$$G_{(r, \mathbb{F})}^r = G^r / G^{r+1} \text{ is } \text{Aut} \left(\left(\mathbb{Q}_{\mathbb{F}}^{(r)} \right) \right)$$

$\begin{array}{c} \text{PE}^{(r)} \\ \parallel \\ (\mathbb{Z}_{\mathbb{F}}^0)_0 \end{array}$

 \uparrow unit section を含む
 逆写像成分

$(\mathbb{Z}_{\mathbb{F}}^0)_0$ $\subset \mathbb{F}$ 上 a smooth 群 scheme

$$0 \rightarrow G^r \rightarrow (\mathbb{Z}_{\mathbb{F}}^0)_0 \rightarrow \mathbb{F} \rightarrow 0 \text{ 完全系列}$$

\downarrow finite étale

$$\begin{array}{c} \mathbb{Q}_{\mathbb{F}}^{(r)} \\ \downarrow \\ \text{PE}^{(r)} \end{array} \text{ a 群 } \alpha \rightarrow \text{Aut} = G$$

\therefore 完全系列の系列 G^r / G^{r+1} は abel 2-p 倍可群 ①
0 可群 である。

補題 F char $p > 0$ の代数閉体

V 有限次元 F -線形空間を F 上の smooth 代数群 G とみたす。 $\text{Spec } S^0 V^*$

G : 有限群

$$1 \rightarrow G \rightarrow E \rightarrow V \rightarrow 1 \text{ } F \text{ 上の群 } G \text{ の完全系列}$$

($G \rightarrow E \rightarrow V$ が群 G の射影 $E \rightarrow V$ (は G 核子))

\Leftrightarrow

(1) E は可換 $\Leftrightarrow pE = 0$

(2) G も可換 $\Leftrightarrow pG = 0$

2) $\text{Hom}_{\mathbb{F}_p}(V, F) \xrightarrow{\cong} \text{Ext}(V, G)$
($\forall G$ は \mathbb{F}_p -同形類群)

(8): $G \xrightarrow{f_i \otimes \delta_i} E$

とすると、

E は f_i は δ_i
 A - S は $\mathbb{F}_p \subseteq E$
 δ_i は T の \mathbb{F}_p -同形

$$0 \rightarrow \mathbb{F}_p \rightarrow E_i \rightarrow V \rightarrow 0$$

$$\parallel \quad \downarrow \quad \downarrow f_i$$

$$0 \rightarrow \mathbb{F}_p \rightarrow G_0 \rightarrow G_0 \rightarrow 0$$

Artin-Schreier
 $\alpha \in \text{Ext}^1$

3) $[E] : \text{Hom}_{\mathbb{F}_p}(G, \mathbb{F}_p) \rightarrow \text{Hom}_{\mathbb{F}_p}(V, F)$ は 同射.
(\mathbb{F}_p -線形)

$$h \longmapsto h_*(E) \quad \forall \mathbb{F}_p \text{ は } \mathbb{F}_p \text{-ext}^1$$

$$0 \rightarrow G \rightarrow E \rightarrow V \rightarrow 0$$

$$\downarrow \quad \downarrow \quad \parallel$$

$$0 \rightarrow \mathbb{F}_p \rightarrow h_*(E) \rightarrow V \rightarrow 0$$

$\text{Hom}(V, F) \xrightarrow{\cong} \text{Ext}(V, \mathbb{F}_p)$
 \downarrow
 $h_*(E)$

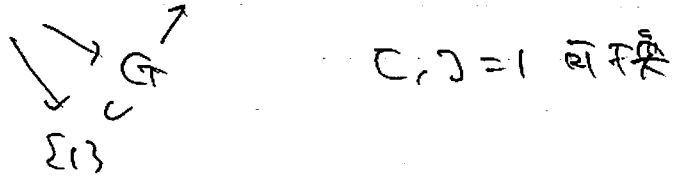
3) 判別同射

rsw: $\text{Hom}(G/G^{rt}, \mathbb{F}_p) \rightarrow \Omega_r(\log D)(R) \otimes_{\mathbb{F}_p} \bar{F}, F = K(\bar{S})$
 $\text{Hom}_{\mathbb{F}_p}(m_K^r / m_K^{rt} \otimes \bar{F}, \Omega_r(\log D) \otimes \bar{F})$

\mathbb{F}_p 判別同射 $e_{p-t} = \frac{1}{T^n}$ rsw = "d $\frac{1}{T^n}$ " = $(T^n F) d(\log \frac{1}{T^n})$
ptn $-nd \log T$

補題の証明 連結 $\rightarrow V \times V \quad [,] = 0$

1) $[,] : E \times E \rightarrow E \rightarrow V$



乗 : $E \rightarrow V \xrightarrow{0} V$
 $E \rightarrow E \xrightarrow{1} V \quad \dots \quad \text{乗} = 1$

2) $V = G_a, G = \mathbb{F}_p \times \mathbb{Z}/2\mathbb{Z}$

$F \rightarrow \text{Ext}(G_a, \mathbb{F}_p)$

$c \mapsto [E_c] \quad \begin{array}{ccccccc} 0 & \rightarrow & \mathbb{F}_p & \rightarrow & E_c & \rightarrow & G_a \rightarrow 0 \\ & & \parallel & & \downarrow & & \downarrow c \text{倍} \\ 0 & \rightarrow & \mathbb{F}_p & \rightarrow & G_a & \rightarrow & G_a \rightarrow 0 \end{array}$

同一形を惹く. 単射

$[E_c] \neq 0 \iff c \neq 0 \iff c \text{倍は同一形, } \exists a \in \mathbb{Z}^2 \text{ } [E_c] \neq 0$

単射 $[E] \in \text{Ext}(G_a, \mathbb{F}_p) \iff E \text{ の種数が } 0 \Rightarrow E \subseteq G_a$

参考文献 1. $\text{Ext}(G_a, G_a)$ Demazure - Gabriel

2. Borel Linear Alg. Group

E の射影影は Jacobian $\rightarrow \delta = 0$

$1 \mapsto C^* \xrightarrow{\text{étale isogeny}}$

$$\begin{array}{l}
 0 \rightarrow \mathbb{F}_p \rightarrow \mathbb{G}_a \rightarrow \mathbb{G}_a \rightarrow 0 \\
 \parallel \downarrow \text{étale} \quad \uparrow \text{étale} \\
 0 \rightarrow \mathbb{F}_p \rightarrow \mathbb{G}_a \rightarrow \mathbb{G}_a \rightarrow 0 \\
 \mapsto 1 \mapsto t \mapsto t^p - t
 \end{array}$$

$\text{deg} = p$
 $f \mapsto at^p + bt$
 $a \neq 0$
 $b \neq 0$
 $a^p + b = 0$ étale

3.12 2. 系存在 a 2 用各

特許種類 \hookrightarrow proper, smooth
 $U \subset X/k$ $\text{ch } k = p > 0$
 open imm.

\mathcal{F} : U 上 (locally constant sheaf of \mathbb{Z}/ℓ^n -module
 $\ell \neq p$)

$\chi_c(U, \mathcal{F})$ Euler 種類

$$\text{Tr } C(j, \mathcal{F}) \leftarrow H^{2d}(X, \mathbb{Z}/\ell^n(d)) \xrightarrow{\text{Tr}} \mathbb{Z}/\ell^n$$

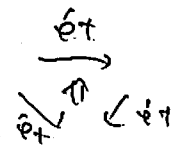
charn class \leftarrow 命定支元 (係, 2 計算 $c_T = 1$)

Lefschetz trace formula (SGA5)

X/k of finite type

$X \in \text{ét}$ obj is X is a étale scheme

mor is X is a morphism (étale)



$X \in \text{ét}$ is a sheaf : 反直觉于 \mathbb{Z}^n $X \in \text{ét} \xrightarrow{\pi} \text{Ens}$ (集合论)

\mathbb{Z}^n
 并不满足条件

$$\{U_i \rightarrow U\}_{i \in I} : \bigcup_i \text{Im}(U_i) = U$$

(= 不成立)

$$\pi(U) \rightarrow \ker(\prod_i \pi(U_i) \rightarrow \prod_i \pi(U_i \times U))$$

is bij.

Y is a scheme $\pi: Y \rightarrow X$, $U \mapsto Y(U)$ is sheaf

$X \in \text{ét}$ is a sheaf π smooth (= l.c.c. (locally constant constructible

is), X is a finite étale scheme \mathbb{Z}^n is not a sheaf.

$$(\text{Finite étale}/X) \xrightarrow{\text{同构}} (\text{l.c.c.}/X \in \text{ét})$$



$(\pi, (x, \pi))$ is a continuous action of a finite group

$X \in \text{ét}$ is a Abelian sheaf (sheaf of Abelian groups)

Abel 群 (Tate, Artin)

$$(A^b / X^b) \rightarrow A^b \quad (\text{Abel 群 } a \text{ 圖})$$

\downarrow
 \neq

\downarrow

$$\longmapsto T(X, \neq) \quad \text{a 導来関手}$$

$H^*(X, \neq)$ 的
定義した。

$$f: X \rightarrow Y/k$$

$$f_* \neq(V) = \hat{\neq}(f^{-1}(V)) \quad \text{"}$$

\downarrow
 $V \subset X$

X 上 a 層 \neq の直像 $f_* \neq$ 的
定義した,

f_* a 正随伴関手 $f^* \leftarrow \text{exact}$.

$$U \hookrightarrow X \xrightarrow{j} Y$$

$$j_* \neq = \ker(j_* \neq \rightarrow (j_* i^* \neq))$$

adjunction $j_* \leftarrow \text{exact}$.

$$X \rightarrow Y \text{ sep}$$

$$\begin{matrix} \text{open} \\ \text{imm.} \\ X \hookrightarrow X \\ \downarrow j \end{matrix}$$

$$\downarrow f \text{ proper}$$

$$R^2 f_* = R^2 f_* \circ j_*$$

X 上 a 層 \neq 的 constructible といは

X a 局所開部分 \neq -a (= 部分層) $X = \coprod_i X_i$

$$j_i: X_i \rightarrow X$$

(locally closed imm)

underlying set a
部分層

$\neq(X_i) = j_{i*} \neq$ 的 l.c.c. (= T 子 E a 的) 存在する

$$\Lambda = \mathbb{Z}/\ell^n \mathbb{Z} \quad \ell \neq \text{char } k$$

$D_{\text{ctf}}(X, \Lambda)$ derived category

↑ constructible (ordimension finite)

obj: X 上 Λ 加群 \mathbb{Z} の層の複体 $\mathcal{F} = (\mathcal{F}^i, d^i)$ 2

整数 $a \leq b$ 2 $\mathcal{F}^i = 0$ 有限 Λ -module $M =$

2 $i \in [a, b]$ 2 0 , $\mathcal{F}^i \cong M$

↑ $i > a$ 2 $\mathcal{F}^i = 0$ \mathcal{F}^i 2 $i \in [a, b]$ 2 0 constructible
 $M \cong$ 自由 Λ 加群 2 分解 $(\mathbb{Z} \otimes)$

mor complex 2 射 modulo $h^1 p$
 quasi-isom. 2 同値性.

6 operation $f: X \rightarrow Y$ k sep. of. fin. type

$$(R) f_* f^! : D_{\text{ctf}}(X, \Lambda) \rightarrow D_{\text{ctf}}(Y, \Lambda)$$

$$f^*(R) f^! : D_{\text{ctf}}(Y, \Lambda) \rightarrow D_{\text{ctf}}(X, \Lambda)$$

↑ Rf_* 2 adjoint
 ↑ $f^!$ 2 adjoint

$$\otimes^L R\text{Hom} : D_{\text{ctf}}(X, \Lambda) \times D_{\text{ctf}}(X, \Lambda) \rightarrow D_{\text{ctf}}(X, \Lambda)$$

$$K_x = Rf^* A \quad f: X \rightarrow Y/k$$

$$f: \text{smooth dim } d \Rightarrow Rf^* A = \Delta(d)[2d]$$

↑ ↑ shift
Tate twist

$$D_x = R\mathcal{H}om(-, K_x) : D_{\text{ctf}}(X, A) \rightarrow D_{\text{ctf}}(X, A)$$