Date 5.24

1. Wild blow-up

2. 分岐群

3. groupoid

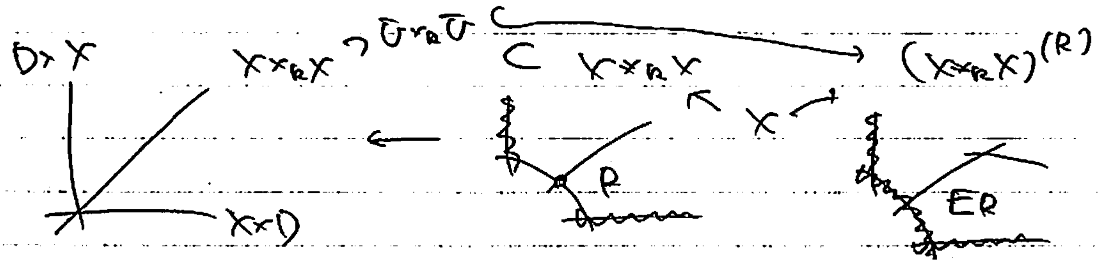
4. 分岐を消す

X = smooth$/k$
$\cup$
$D = \bigcup D_i$ simple normal crossing divisor

$R = \sum r_i D_i \quad r_i \in \mathbb{N}$

$R \hookrightarrow X \to X \times_k X$ log diagonal

blow-up して $(X\times_k X)\times_X R$ の proper transform を除く



$E_R \subset (X\times_k X)^{(R)} \supset U\times_k U$

exceptional divisor　　open

$(X\times_k X)^{(R)} = U\times_k U$ となるのは $\forall i,\ r_i > 0$

例　$X = \mathbb{A}^d_k = \operatorname{Spec} k[T_1,\dots,T_d] \supset D = (T_1\cdots T_n)$

$X\times_k X = \operatorname{Spec} k[U_1^{\pm 1},\dots,U_n^{\pm 1}, S_{n+1},\dots,S_d, T_1,\dots,T_d]$

$U_i = \dfrac{S_i}{T_i}$　　　$\overset{\shortparallel}{A}$

$X \hookrightarrow X \times_R X \qquad U_i = 1 \quad S_j = T_j$

$R \subset X \qquad T^R = T_1^{r_1} \cdots T_n^{r_n} \quad U_i = 1 + T^R V_i \quad S_j = T_j + T^R V_j.$

$$(X \times_R X)^{(R)} = \operatorname{Spec} A\left[\underbrace{\frac{U_1 - 1}{T^R}}_{V_1}, \ldots, \frac{U_n - 1}{T^R}, \frac{S_{n+1} - T_{n+1}}{T^R}, \ldots, \underbrace{\frac{S_d - T_d}{T^R}}_{V_d}\right]$$

$$= \operatorname{Spec} B\left[V_1, \ldots, V_d, T_1, \ldots, T_d, \frac{1}{1 + T^R V_1}, \ldots, \frac{1}{1 + T^R V_n}\right]$$

B上 smooth 次元 $2d$.

$\cup$

$U \times_R U \qquad T_1, \ldots, T_n$ を可逆

$X \to (X \times X)^{(R)} \quad (V_1, \ldots, V_d)$

2. $\mathcal{O}_L \leftarrow \mathcal{O}_K[x_1, \ldots, x_n]/(f_1, \ldots, f_n)$

$$\begin{tikzcd}
T = \operatorname{Spec} \mathcal{O}_L \arrow[r] \arrow[d] \arrow[dr, phantom, "\circlearrowleft"] & \mathbb{A}^n_T \arrow[d, "f"] & \arrow[d, phantom, "\rightsquigarrow"] & \coprod_{\sigma \in G} s' \arrow[r] \arrow[d] & \operatorname{Spec} \hat{Q}^{(r)}_{s'} \arrow[d] \\
S = \operatorname{Spec} \mathcal{O}_K \arrow[r, "0\text{-Section}"'] & \mathbb{A}^n_S & & s' \arrow[r] & \operatorname{Spec} \hat{P}^{(r)}_{s'}
\end{tikzcd}$$

r, s'

$G_T \to \pi_0(\operatorname{Spec} \hat{Q}^{(r)}_{s'}) = \pi_0(\operatorname{Spec} Q_F^{(}$

$G \to (\mathrm{Spec})\, Q_{F'}^{(r)}$

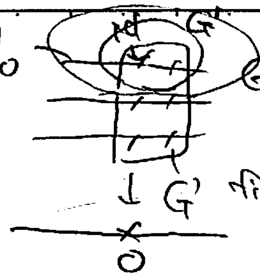
$\downarrow$

$(\mathrm{Spec})\, P_{F'}^{(r)} = \mathbb{A}_{F'}^{n}$

$G^{r+} = \{1\}$

この図式にGが作用しているかのように思うことができる.

$(Q_{F'}^{(r)})_0$



$Q_{F',0}^{(r)} \to \mathbb{A}_{F'}^{n}$

連結な étale 被覆 revêtement

finite étale

G 捻子・主等質空間

有限群

S : scheme $X \to S$ finite étale surjective

Gの作用 XにS上作用

$\coprod_{\sigma \in G} X = G \times X \longrightarrow X \times_S X$ が同形

$(\sigma, x) \longmapsto (\sigma x, x)$

$G \curvearrowright Q_{F'}^{(r)} \to P_{F'}^{(r)} = \mathbb{A}_{F'}^{n}$ G-torsor ($\Leftarrow G^r$ が abel)

stabilizer

$G^r \curvearrowright Q_{F',0}^{(r)} \subset Q_{F'}^{(r)}$

捻子 連結

$\pi_1(\mathbb{A}_{F'}^{n}, 0) \twoheadrightarrow G^r = G^r / G^{r+}$

分岐群の次数商は $\mathbb{A}^n$ の基本群の商.

No.

Date

$Q^r_{F',0}$ を $\mathbb{A}^n_{F'}$ と同形，で $\mathbb{A}^n_{F'} \to \mathbb{A}^n_{F'}$

$f_1, \dots, f_n \in F'[T_1, \dots, T_n]$

$\uparrow$

$T_1, \dots, T_n, T_1^p, \dots, T_n^p, \underline{T_1^{p^2}, \dots, T_n^{p^2}, \dots}$

の1次結合．↗ うちきでおくる評価

となることを示すのが当面の目標

$$\begin{array}{ccccc} \mathrm{Spec}\,\mathcal{O}_L & & & & \\ \| & & & & \\ T & \to & Y & \supset E = Y \setminus V & \leftarrow V\text{での整閉包} \\ \downarrow & \square & \downarrow & \uparrow & \\ S & \to & X & \supset D & \leftarrow \text{smooth, 既約} \\ \| & & & \downarrow & \\ \mathrm{Spec}\,\mathcal{O}_K & & & \xi & \text{generic point} \end{array}$$

$V \xrightarrow{\text{finite étale, } G\text{-torsor}} U = X \setminus D$ smooth

$\mathcal{O}_K = \hat{\mathcal{O}}_{X,\xi}$

L/K 有限次 Galois $G = \mathrm{Gal}(L/K)$

$$\begin{array}{ccccc} Y & \to & Y \times_k X = Q & \supset & V \times_k U \\ \downarrow & \square & \downarrow & \square & \downarrow \\ X & \to & X \times X = P & \supset & U \times_k U \\ & & \downarrow pr_2 & & \\ S & \to & X & & \end{array}$$

finite flat

で base change

$V \times_k U$ の中での P の整閉包．

$$\begin{array}{ccc} T & \to & Q_S \\ \downarrow & & \downarrow \\ S & \to & P_S = P \times_k S = X \times S \\ & & \downarrow \\ S' & & S \end{array}$$

log つき G in smooth

finite flat

pr_2 の base change

$\mathrm{Spec}\,\mathcal{O}_{F'}/m_{F'}^a$

smooth

log なし

$$\begin{array}{ccc} T & \to & \mathbb{A}^n_S \\ \downarrow & & \downarrow \\ S & \to & \mathbb{A}^n_S \\ & & \downarrow \\ & & S \end{array}$$

smooth

$Q_{S'}^{(r)}$

$\downarrow$

$P_{S'}^{(r)}$

log なしがほしければ $X \times_k X$ の

代わりに $X \times_k X$ を使えばよい．

この方法の利点：今の図式にGが作用する。

利点を生かしきっていない。

$$(S,T,W) \longmapsto (S,W)$$

3. $$\begin{array}{ccc}
(X\times_k X)\times_X (X\times_k X) = X\times X\times X & \xrightarrow{pr_{1,3}} & X\times X \\
 & & \cup \\
 & 亜群 \quad \curvearrowright & U_{Xa}U \quad \text{dense open.} \\
\uparrow & & \cap \\
(X\times X)^{(R)}\times_X (X\times X)^{(R)} & \xrightarrow[\exists !]{} & (X\times X)^{(R)}
\end{array}$$

↑ レポート：たしかめる。

<u>例</u>

$$X = \mathbb{A}^1 = \operatorname{Spec} k[T] \supset D = (T)$$

$$R = rD$$

$$\left(\frac{S}{W}\right)^{\pm 1},\quad \frac{\left(\frac{S}{W}-1\right)}{W^r} \qquad (X\times_k X)^{(R)} = \operatorname{Spec} k[S, W, 左の2つ]$$

↙?

$(X\times X)^{(R)}\times(X\times X)^{(R)} = \operatorname{Spec} k[S, T, W, (S, Tに対する左の2つ), (T, Wに対する左の2つ),$

$$\left(\frac{S}{W}\right)^{\pm 1} = \left(\left(\frac{S}{T}\right)\left(\frac{T}{W}\right)\right)^{\pm 1},\quad \frac{\frac{S}{W}-1}{W^r} = \frac{S}{T}\,\frac{\frac{S}{W}-1}{W^r} + \frac{\frac{S}{T}-1}{T^r}\cdot\left(\frac{T}{W}\right)^r$$

No.

Date

$X \to (X \times_k X)^{(R)}$

普遍性の公理 — $U \times U$ に制限して確かめればよい。

$$
\begin{tikzcd}
(X \times_k X)^{(R)} \arrow[r, shift left, "pr_1"] \arrow[r, shift right, "pr_2"'] & X \\
\cup & \cup \\
E_R \arrow[r] & R
\end{tikzcd}
$$

— vector束

pr₁とpr₂の制限が一致

$\|$

$\Theta^{(R)} = \mathbb{V}(\Omega^1_X(\log)(R)) \times_X R$ $\qquad \Omega^1_X(\log D) = N_{X/X\times X}$

twisted log tangent b'dle

$\Omega^1_X(\log D)(R) = \Omega^1_X(\log D) \otimes_{\mathcal{O}_X} \mathcal{O}_X(R)$

$= N_{X/(X\times X)^{(R)}}$

$\mathbb{V}(\mathcal{E}) = \operatorname{Spec} S^\bullet_{\mathcal{O}_X} \mathcal{E}$

X上の可変ベクトル束

$$
\begin{array}{ccc}
(X\times X)^{(R)} = \operatorname{Spec} k[V, T, \frac{1}{1+VT^r}] & \longrightarrow & X = \operatorname{Spec} k[T] \\
\cup & & \cup \\
E^{(R)} = \operatorname{Spec} k[T]/(T^r)[V] & \longrightarrow & R = \operatorname{Spec} k[T]/(T^r)
\end{array}
$$

$$
\begin{array}{ccc}
Y \times Y & \supset & \underbrace{V \times V} \\
\downarrow & & \downarrow \\
X \times X & \supset & U \times U
\end{array}
$$

$$
\begin{tikzcd}
 & & V & \operatorname{Spec} L \arrow[l] & \\
U \times U \arrow[d, "pr_2"] & & \Delta & & \\
U & & \operatorname{Spec} K \arrow[ll] & \operatorname{Spec} K' \arrow[l] \arrow[uu]
\end{tikzcd}
$$

$$
\begin{array}{cc}
\Theta_S & \Theta_{S'}^{(r)} \\
\downarrow & \downarrow \\
P_S & P_{S'}^{(r)}
\end{array}
$$

S'

$V\times\bar U$ の代わりに

$$\underbrace{(V\times \bar U)}_{pr_2}\times_{\bar U} V = あ \times_{\bar U} V$$

となるものでおきかえても構わない.

$$V\times \bar U = (V\times V)/(1\times G) \qquad (V\times\bar U)\times_{\bar U} V = V\times V$$

$$W = (V\times V)/\Delta G \qquad W\times_{\bar U} V = V\times V$$

$$\begin{array}{ccc} W\times_{\bar U} W & \longrightarrow & W \\ \downarrow & & \downarrow \\ \underbrace{(\bar U\times\bar U)}_{pr_2}\times_{\bar U}\underbrace{(\bar U\times\bar U)}_{pr_1} & \longrightarrow & \bar U\times\bar U \quad pr_{1,3} \end{array}$$

$$W\times_{\bar U} W = V\times V\times V/\Delta G$$

$G\times G\times G \supset \Delta G\times G$ による商 $W\times_{\bar U}\underbrace{(\bar U\times\bar U)}_{pr_1}$

$G\times\Delta G$ による商 $\underbrace{(\bar U\times\bar U)}_{pr_2}\times_{\bar U} W$

$$\begin{array}{ccc} W\times_{\bar U} W & = & V\times V\times V/\Delta G \\ \downarrow & & \downarrow pr_{13} \\ W & = & V\times V/\Delta G \end{array}$$

No.

Date

$$\begin{tikzcd} V/G \arrow[r, "\Delta_V"] \arrow[d, equal] & V\times V/\Delta G \arrow[d, equal] \\ U \arrow[r] & W \end{tikzcd}$$

単位元 ····> $W \supset U$ は亜群

$$\begin{tikzcd} Z^{(R)} \arrow[r, hook] \arrow[d] & W \arrow[d, "\leftarrow \text{finite étale}"] \\ (X*X)^{(R)} & U\times U \arrow[l, hook'] \end{tikzcd}$$

$(X*X)^{(R)}$ の W での整閉包.

$Z^{(R)} \to (X*X)^{(R)}$ が étale になるような例

$X = \mathbb{A}^1 \supset U = \mathbb{G}_m = \mathrm{Spec}\, k[T^{\pm1}]$

$\mathbb{A}^1 = \mathrm{Spec}\, k[T]$ $\qquad U\times U = \mathrm{Spec}\, k[S^{\pm1}, T^{\pm1}]$

1. $V \to U$ $\quad t^n = T$ $\quad p \nmid n$ Kummer 被覆

$W \to U\times U$ $\quad t^n = S\cdot T^{-1}$ $\quad (V\times V \quad s^n = S,\ t^n = T)$

$$\begin{array}{ccc} W & \to & U\times U \\ \cap & & \uparrow \\ Z & \to & X*X \end{array}$$

$Z \to X*X = \mathrm{Spec}\, k[T, U^{\pm1}]$ $\quad U = \frac{S}{T}$ $\quad t^n = U$

finite étale

2. $V \to U$ $\quad t^p - t = \frac{1}{T^n}$ $\quad p \nmid n$ Artin-Schreier

$W \to U \times U$. $\quad t^p - t = \frac{1}{S^n} - \frac{1}{t^n}$

$R = nD$ $\quad D = V(T)$

$$(X * X)^{(R)} = \mathrm{Spec}\, R[V, T, \underline{\tfrac{1}{1+VT^n}}] \qquad \frac{S}{T} = 1 + VT^n$$

$$\frac{1}{S^n} - \frac{1}{T^n} = \frac{1}{T^n}\left(\left(\frac{1}{1+VT^n}\right)^n - 1\right)$$

$= nVT^n + T^{2n}(\cdots$

$$= \underline{\frac{1}{(1+VT^n)^n}} \, \frac{1-(1+VT^n)^n}{\cancel{T^n}}$$

$$E_R \subset (X * X)^{(R)} \supset U \times U$$

$T^n = 0$

$\underline{\mathrm{Spec}\, k[T]/(T^n)}\,[V]$

$$t^p - t = -nV$$

$$V = \frac{\frac{S}{T} - 1}{T^n}$$

$$\frac{S}{T} - 1 = d\log T$$

$X : t^p - t = \frac{1}{T^n}$

$$t^p - t = -n\frac{d\log T}{T^n} = d(\log T^{-n}) = \mathrm{rsw}\, X$$

$$Z_R = \mathrm{Spec}\, k[T]/(T^n)[\cancel{V}][t]/(\cancel{t^p - t} + nV) = A^1_R$$

$$Z_R = A^1_R \to A^1_R = E_R$$

$$-\frac{1}{n}(t^p - t) \leftarrow V$$