3.§3 分岐理論と特性サイクル

X smooth $\supset D$ irred div. $\ni \xi$ gen pt

$$K = \mathrm{Frac}\, \mathcal{O}^{sh}_{X,\xi}$$

$V \to U = X - D$ G-torsor

$\mathcal{F}$. $\mathcal{G}$ l.c.c on U. trivialized on V. corresponding to rep. M of G

$I \subset G$ inertia subgp.

$G^r \lhd I$ filtration by ram gp $r \in \mathbb{Q}, \geq 1$.

$\exists$ $1 = r_0 < r_1 < \cdots < r_n$ rational. s.t.
G^r は $(r_{i-1}, r_i]$, (r_n, ∞) で一定. $I = G^1$. $G^{r_n+} = 1$

$$\mathrm{Gr}^{r_i} G = G^{r_i}/G^{r_i+} = \begin{cases} p\text{と素 cyclic} & i = 0 \\ \mathbb{F}_p\text{-vect.sp} & > 0 \end{cases}$$

$r > 1$

$$\mathrm{char}: \mathrm{Hom}_{\mathbb{F}_p}(\mathrm{Gr}^r G, \mathbb{F}_p) \to \mathrm{Hom}(\mathfrak{m}^r_{\bar K}/\mathfrak{m}^{r+}_{\bar K}, \Omega^1_{X,\xi} \otimes \bar F) \quad \text{標準単射}$$
$$\chi \longmapsto \mathrm{char}(\chi)$$

$\bar K = K_{sep}$. $\bar F$ res. fld of $\bar K$ = alg closure of $k(\xi)$

slope decomposition
$$M = \bigoplus_{r \geq 1} M^{(r)} \qquad M^{G^r} = \bigoplus_{s<r} M^{(s)}$$

$\mu_p \subset \Lambda^\times$, $\mu_p \cong \mathbb{F}_p$
$$M^{(r)} = \bigoplus_{\chi: \mathrm{Gr}^r G \to \Lambda^\times} \chi^{\oplus m(\chi)} \qquad \text{rep of } \mathrm{Gr}^r G = G^r/G^{r+}$$

$$\mathrm{char}\,\chi: \mathfrak{m}^r_{\bar K}/\mathfrak{m}^{r+}_{\bar K} \to \Omega^1 \otimes \bar F$$

$$\underset{\text{line bdle on } D_\chi}{L_\chi} \longrightarrow T^*X \times_X D_\chi$$

$\pi_\chi: D_\chi \to D$ finite on a nbd of ξ.

定理 3.1 ξ の étale 近傍で次が成り立つ.

1. $SS(j_!\mathcal{G}) = T^*_X X \,(\cup\, T^*_D X \text{ if } M^{(1)} \neq 0) \cup \bigcup_{r>1} \bigcup_{x} \mathrm{Im}(\mathrm{char}_x : L_x \to T^*X)$

2. $CC(j_!\mathcal{G}) = (-1)^n \Big(\mathrm{rk}\,\mathcal{G} \cdot [T^*_X X] + \mathrm{rk}\, M^{(1)} \cdot [T^*_D X] + \sum_{r>1} \sum_{x} r \cdot m(x) \dfrac{\pi_{x*}[L_x]}{[D_x : D]} \Big)$

証明. 1. ・char の transversal な pull-back との compatibility

・Swan conductor の semi-continuity (loc. const ⇒ loc. acyclic)

⊃ ・2 + SS = supp CC.

2. 定理1.2 で curve に帰着. (cutting-by-curve!)

定理1.2 の証明に定理3.1 の dim X=2 の場合が必要.

$C \subset T^*X$ $\dim C_a = n$. $A = \sum m_a C_a$. $D(A) = \pi_* \mathbb{P}(A)$

$\xi \in D \subset X$ $D(A) = \sum m_a [\mathbb{P}(C_a) : D] \cdot D$ ~~$= \pi_*\mathbb{P}(A)$~~

命題 3.2 $X \subset \mathbb{P}$ proj smooth surface. ~~D div~~

$\chi_c(X, \mathcal{F}) - (CC(\mathcal{F}), T^*_X X)$ D(右辺)

$= -2 \cdot (\mathrm{Art}(\mathcal{F}) - D(CC(\mathcal{F})), H)$

系 1 Art$\mathcal{F}$ は $D(CC\mathcal{F})$ と数値的同値.

2 $\chi_c(X, \mathcal{F}) = (CC(\mathcal{F}), T^*_X X)$

定理3.1.2 の証明 $[L_x]$ の係数. étale local additive について.

cpt $X \leftarrow X' \leftarrow Y$ num eq. ⇒ 係数の一致.