$$\chi(X,\mathcal{F}) = \chi(X_L, \pi^*\mathcal{F}) - \chi(V, \mathcal{F}|_V)$$

↑ 帰納法の仮定＋定理1.3

$$\chi(X_L, \pi^*\mathcal{F}) = \chi(L, Rp_{L*}\pi^*\mathcal{F})$$

$$\underset{\text{G-O-S}}{=} \underset{2}{\underset{\shortparallel}{\chi(L)}} \cdot \chi(W, \mathcal{F}|_W) - \sum_u \dim\mathrm{tot}\,\Phi_u(\mathcal{F}, p_L)$$

Milnor公式

11/6 指数公式

$$\begin{tikzcd} W \arrow[rr, bend left, "i"] \subset X' \arrow[r, "\pi"] \arrow[d, "p_L"'] & X \supset V \arrow[d] \\ y \in L & \end{tikzcd}$$

π: blow-up along V, $i' : V \to X$

$$\chi(X,\mathcal{F}) = \chi(X_L, \pi^*\mathcal{F}) - \underline{\chi(V, \mathcal{F}|_V)}$$

$$\chi(X_L, \pi^*\mathcal{F}) = 2\cdot \underline{\chi(W, \mathcal{F}|_W)}$$

$$- \sum_u \underline{\dim\mathrm{tot}\,\Phi_u(\mathcal{F}, p_L)}$$

↗ 左辺

示したいこと: $\chi(X,\mathcal{F}) = (CC\,\mathcal{F}, T^*_X X)_{T^*X}$

∵ $\chi(V,\mathcal{F}|_V)$, $\chi(W,\mathcal{F}|_W)$ に帰納法の仮定.

$\dim\mathrm{tot}\,\Phi_u(\mathcal{F}, p_L)$ に Milnor公式を使う.

右辺 $A = CC\,\mathcal{F}$

$$(A, T^*_X X) = (\pi^! A, T^*_{X'} X')_{T^*X'} - (i'^! A, T^*_V V)_{T^*V}$$

$$(\pi^! A, T^*_{X'} X') = 2\cdot (i^! A, T^*_W W) + \sum_u (A, dp_L)$$

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補題 2.2 $A = \sum m_a C_a$, $C = \bigcup_a C_a$

1. $f : X \to Y$ X: proper smooth, Y: proper smooth curve genus g

f: proper flat, Y の dense open 上 smooth

$W \subset X$ general fiber $i : W \to X$ properly C-tra

f は C に関して isolated char pt しかもたない

$$(A, T^*_X X)_{T^*X} = (2-2g)(i^! A, T^*_W W)_{T^*W} + \sum_{\substack{u:\text{isol.}\\ \text{char. pt}}} (A, df)_{T^*X, u}$$

2. $i' : V \to X$ closed imm V: codim 2 smooth

i': properly C-trans

$\pi : X' \to X$ blow-up (properly C-trans)

$$(\pi^! A, T^*_{X'} X')_{T^*X'} = (A, T^*_X X)_{T^*X} + (i'^! A, T^*_V V,$$

証明 1. $f = \mathrm{id}$ $X = Y$

$C \subset T^*Y$ C_a: 0-section or fiber.

$A = C_a$ fiber の場合 $1 = (2-2g) \cdot 0 + 1$

0-section の場合 $\deg \Omega^1_Y = (2-2g)(-1) + 0$

$i^!$ で $(-1)^{\dim X - \dim}$ がかかる.

f：一般の場合.

$f_! A \leftarrow df^! A$ の push-forward.

$T^*Y \leftarrow X \times_Y T^*Y \xrightarrow{df} T^*X$

$T^*_Y Y$

$$\begin{tikzcd} W \arrow[r, "i"] \arrow[d] & X \arrow[d] \\ y \arrow[r, "i_y"] & Y \end{tikzcd}$$

$(f_! A, T^*_Y Y)_{T^*Y} = (2-2g)(i_y^! f_! A, T^*_y y)_{T^*y}$

$\| \leftarrow$ proj. formula

$(df^! A, X \times_Y T^*_Y Y)_{X \times_Y T^*Y}$ $\quad + \sum (f_! A, d(id))_{T^*Y, v}$ $\quad v = f(u)$

$\|$

$(A, T^*_X X)_{T^*X}$ 右辺も同様

2. $T^*X' \xleftarrow{d\pi} X' \times_X T^*X \xrightarrow{\pi \times 1} T^*X$ $\quad (X' \times_X T^*_X X) + K$

$(\pi^! A, T^*_{X'} X')_{T^*X'} = ((\pi \times 1)^! A, d\pi^! (T^*_{X'} X'))$

$$\begin{tikzcd} E \arrow[r, phantom, "\subset"] \arrow[d] & X' \arrow[d] \\ V \arrow[r] & X \end{tikzcd}$$

$\mathbb{P}^1$-bdle

$$\begin{tikzcd}
 & 0 \arrow[d] & 0 \arrow[d] & & \\
0 & T^*_E X' \arrow[l] \arrow[d] & T^*_V X \times_V E \arrow[l] \arrow[d] & K \arrow[l] & 0 \arrow[l] \\
 & T^*X' \times_{X'} E \arrow[d] & T^*X \times_X E \arrow[l] \arrow[d] & & \\
0 & T^*(E/V) \arrow[l] & T^*E \arrow[l] \arrow[d] & T^*V \times_V E \arrow[l] \arrow[d] & 0 \arrow[l] \\
 & & 0 & 0 &
\end{tikzcd}$$

K: rk2 $\quad$ E上の line bdle

$T^*(E/V)$: line bdle

$$(\pi^! A, T^*_{X'} X')_{T^*X'} = (\underbrace{(\pi\times 1)^! A, X'\times_X T^*_X X}_{(A,\, T^*_X X)} + K)_{X'\times_X T^*X}$$

$$((\pi\times 1)^! A, K) = (i^! A, T^*_V V)$$

$$\underset{\text{excess int. formula}}{=} (i^! A, T^*_V V)\cdot \underbrace{\deg(T^*_E X)}_{= +1} \qquad E \to V : \mathbb{P}^1\text{-bdle}$$

ひき戻しの公式（定理1.3）⇒ 指数公式（定理2.1）

3.3 分岐理論と特性サイクル

X/k smooth k: perfect.

$D \subset X$ irred div. $\xi \in D$ gen. pt.

$K = \mathrm{Frac}(\mathcal{O}^{sh}_{X,\xi})$ 局所体

strictly local dvr

residue field $k(\xi)$ = function field of D

$U = X - D$

$V \to U$ G-torsor G 有限群

$I \subset G$ inertia subgp $=$ ξ'の上にある geom. pt の固定部分群

$$\begin{array}{ccccc} \xi' & \in & Y & \leftarrow & V \\ \downarrow & & \downarrow & & \downarrow \\ \xi & \in & X & \leftarrow & U \end{array}$$

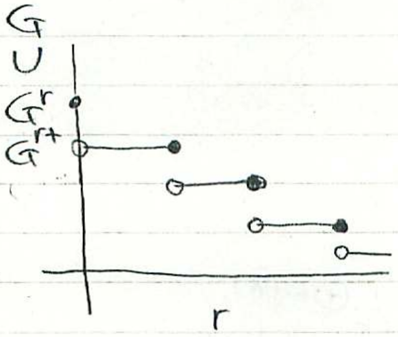
減少 filtration

$G^r \triangleleft I$ 分岐群の filtration $r \in \mathbb{Q}_{\geq 1}$

$G^1 = I$ $r \gg 1$ なら $G^r = \{1\}$

$\exists\, 1 = r_0 < r_1 < \cdots < r_n \in \mathbb{Q}$

$(r_{i-1}, r_i]$ $i=1,\dots,n$, (r_n, ∞) で G^r は一定.



$G^{r_i+} = G^{r_{i+1}}$

$Gr^r G = G^r / G^{r+}$

$Gr^r G = \begin{cases} p\text{ と素な巡回群} & r = 1 \\ \mathbb{F}_p\text{-vect. space} & r > 1 \end{cases}$

$G^{1+} = P \subset I$: p-Sylow 群

$r > 1$ 標準単射

$$\begin{array}{ccc} \mathrm{Hom}_{\mathbb{F}_p}(Gr^r G, \mathbb{F}_p) & \hookrightarrow & \mathrm{Hom}(m_{\bar K}^r / m_{\bar K}^{r+}, \underbrace{\Omega^1_{X,\xi} \otimes \bar F}_{n\text{次元 } \bar F\text{-vect.sp}}) \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ \chi & \longmapsto & \mathrm{char}(\chi) \end{array}$$

$\bar K = K_{sep}$ $m_{\bar K}^r = \{ a \in \bar K \mid \mathrm{ord}(a) \geq r \}$

$m_{\bar K}^{r+} = \{ 〃 \mid 〃 > 0 \}$

代数閉

$\bar F = \bar K$ の剰余体 $= m_{\bar K}^0 / m_{\bar K}^{0+}$

$m_{\bar K}^r / m_{\bar K}^{r+}$ 1次元 $\bar F$ 線形空間

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$\mathcal{G}$: U上の l.c.c. sheaf $\mathcal{G}|_V$ const

$\updownarrow$

G の表現 (Λ-module) M

M の slope decomposition (Λ $\mathbb{Z}_\ell$-alg. $G^{1+} = P$ p群, $p \neq \ell$)

$$M = \bigoplus_{r \in \mathbb{Q}_{\geq 1}} M^{(r)} \quad \text{characterized by}$$

$$M^{G^{r+}} = \bigoplus_{s \leq r} M^{(s)}$$

$\uparrow$

G^{r+} 不変部分

$$\mu_p \subset \Lambda^\times \qquad M^{(r)} = \bigoplus \chi^{\oplus m(\chi)}$$

$\wr\|$ $\circlearrowright$ $\chi : Gr^r G \to \mathbb{F}_p \simeq \mu_p \subset \Lambda^\times$

$\mathbb{F}_p$ $Gr^r G = G^r / G^{r+1}$

$m(\chi) \neq 0$ char $\chi : m_K^r / m_K^{r+} \to \Omega \otimes \bar{F}$

$\neq$

0

$$\begin{tikzcd}
\text{line bdle} \arrow[r, phantom, "\subset"] & L_\chi \arrow[r, hook, "\text{char}\,\chi"] \arrow[dr] & T^*X \times_X D_\chi \arrow[d] \arrow[r] & T^*X \\
 & & D_\chi &
\end{tikzcd}$$

$\pi_\chi : D_\chi \to D \subset X$

ξ の近傍上の有限射

(char χ の像の係数を全て含むような被覆)

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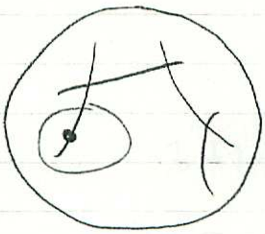
定理 3.1 $\mathcal{G} \neq 0$

$\bar{x}$ ($\bar{x}$ の上の geom. pt) の étale 近傍で次が成り立つ

1. $SS(j_!\mathcal{G}) = T^*_X X \;(\cup T^*_D X) \cup \bigcup_{r>1} \bigcup_{\chi} \mathrm{Im}(\mathrm{char}\,\chi)$

   (if $M^{(1)} \neq 0$) $\qquad$ ($m(\chi) \neq 0$)

2. $CC(j_!\mathcal{G}) = (-1)^n \Big(\mathrm{rk}\,\mathcal{G}\,[T^*_X X] + \mathrm{rk}\,M^{(1)}\,[T^*_D X] + \sum_{r>1}\sum_{\chi} r \cdot m(\chi) \dfrac{\pi_{\chi *}[L_\chi]}{[D_\chi : D]}\Big)$



$\Big(r \cdot m(\chi) \cdot \dfrac{1}{[D_\chi : D]} \in \mathbb{Z}[\tfrac{1}{p}] \Big)$

例 $\mathbb{A}^2$ $D = (x = 0)$ $\mathcal{G} : t^p - t = \dfrac{y}{x^p}$ のとき

$(-1)^2 \Big(1 \cdot [T^*_X X] + 0 \cdot [T^*_D X] + 1 \cdot p \cdot \dfrac{\langle dy \rangle}{1}\Big)$

$(\langle dy \rangle \subset T^*X \times_X D)$

証明

1. (c) 右辺を C としたとき C-transversal な curve への射 $X \to Y$ の local acyclicity を示す.

- Swan conductor の semi-continuity (Deligne–Laumon)

(locally constant $\Rightarrow$ locally acyclic)

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$$\begin{array}{ccc} X & \to & S \\ \cup & & \downarrow \\ X_s & \to & s \end{array}$$

・ rel. dim 1. smooth

transversal ⇒ pull-back との可逆の compatibility

分岐理論に出てくる積の blow-up の functoriality

(⊃) 2 + $j_! \mathcal{F}$: perverse sheaf のとき SS = Supp CC

2. 定理 1.2 を使う.

1 の (⊂) より係数をくらべればよい.

CC の加法性より $\mathcal{L}[x]$ が 1つだけとしてよい.

curve に帰着

$Y \to X$: C-transversal な imm. による pull-back

smooth curve

1 の可逆

定理 1.2 の証明に定理 3.1 の $\dim X = 2$ の場合が必要.

ここを示せばよい.