Tokyo-Berkeley Mathematics Workshop
Partial Differential Equations and Mathematical Physics

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Graduate School of Mathematical Sciences, Univeristy of Tokyo

Abstracts

Lectures:

S. Dyatlov, M. Zworski: Microlocal methods in chaotic dynamics

Following the insights of Faure-Sjöstand and Tsujii, microlocal/semiclassical methods have proved themselves very useful in the study of closed and open smooth hyperbolic systems. We will explain how they give a simple proof of the meromorphy of Ruelle zeta function for Anosov flow (and indicate the ideas behing the Axiom A case), show stochastic stability of Ruelle resonances, and explain the order of vanishing of the zeta function at zero in the case of negatively curved surfaces (showing in particular that the length spectrum determines the genus).

P. Hintz: Non-linear stability of Kerr-de Sitter black holes

I will describe some of the key ingredients of the proof of the non-linear stability of slowly rotating Kerr-de Sitter black holes as solutions to Einstein's field equations, obtained in joint work with András Vasy. I will describe the geometry of these spacetimes, the regularity and asymptotic behavior of linear waves on them, aspects of the non-linear analysis, and how to deal with the diffeomorphism invariance of Einstein's equations.

K. Ito: Stationary scattering theory on manifolds with ends

We discuss the stationary scattering theory on manifolds with asymptotically Euclidean and/or hyperbolic ends. Our main steps are Rellich's theorem, limiting absorption principle, radiation condition, generalized Fourier transform and stationary scattering matrix. We intensively use a commutator argument.
This talk is based on joint works with Erik Skibsted, Aarhus University.

T. Yoneda: Mathematical considerations of laminar-turbulent transition and vortex thinning in 2D turbulence

The dynamics along the particle trajectories for the 3D axisymmetric Euler equations are considered. It is shown that if the inflow-outflow is rapidly increasing in time, the corresponding laminar profile of the Euler flow is not (in some sense) stable provided that the swirling component is not zero. This exhibits a laminar-turbulent transition in pure mathematics. In the proof, Frenet-Serret formulas and orthonormal moving frame are essentially used.

If time remains, then we also discuss vortex thinning process in two dimensional turbulence. This topic is closely related to the recent breakthrough by Bourgain and Li (2015).

Seminar Talks:

K. Abe: Global well-posedness of the two-dimensional exterior Navier-Stokes equations for non-decaying data

We consider the two-dimensional Navier-Stokes equations in an exterior domain, subject to the non-slip boundary condition. It is well known that there exist various non-trivial stationary solutions, which are asymptotically constant and with a finite Dirichlet integral. On the other hand, even local solvability was unknown for the non-stationary problem for such non-decaying initial data. In this talk, we report some global well-posedness result for bounded initial data with a finite Dirichlet integral, and existence of asymptotically constant solutions for arbitrary large Reynolds numbers.

A. Drouot: Pollicott-Ruelle resonances via kinetic Brownian motion

The geodesic flow on the cosphere bundle of a negatively curved manifold has the Anosov property. Hence, the associated classical corellations decay exponentially with quantized rates called Pollicott-Ruelle resonances. We interpret these (dynamical) quantities as probabilistic objects. More precisely, we show that the Pollicott-Ruelle resonances are limits of eigenvalues of the generator of kinetic Brownian motion, which is a random perturbation of the geodesic flow.

F. Macia: Concentration, non-concentration and controllability of Schrödinger flows

Y. Miyamoto: Intersection number and applications for semilinear elliptic equations with general supercritical growth

We study radial solutions of the semilinear elliptic equation $\Delta u + f(u) = 0$ under rather general growth conditions on f. We construct a radial singular solution and study the intersection number between the singular solution and a regular solution. Three applications of the intersection number are given: the Morse index of the singular solution, the bifurcation diagram of an elliptic Dirichlet problem in a ball, and the Type I blow-up solution of a parabolic problem. To this end, we derive a certain limit equation from the original equation at infinity, using a generalized similarity transformation.

H. Mizutani: Global-in-time Strichartz estimates for Schrödinger equations on long-range asymptotically conic manifolds

We will discuss a recent result on Strichartz estimates for Schrödinger equations on long-range asymptotically conic manifolds. Under the condition that the set of trapped trajectories is empty or sufficiently filamentary, we show that the solution satisfies global-in-time Strichartz estimates without loss of derivatives. As an application, we also prove small dada scattering for the mass critical nonlinear Schrödinger equation. The main new difficulty stems in the analysis of low frequencies since, due to the uncertainty principle, low frequency data cannot be studied purely by microlocal techniques. We develop a new version of the Isozaki-Kitada parametrix construction outside the uncertainty region, while we use (homogeneous) Sobolev estimates and low frequency resolvent estimates with sharp weight inside the uncertainty region. This talk is based on joint work with Jean-Marc Bouclet (Toulouse III).

M. Tsujii: The spectrum of semi-classical transfer operator for expanding-semi flows

In this talk, we consider expanding semi-flows given as suspensions of angle multiplying maps on the circle and discuss about the spectrum of associated transfer operators. Suppose that we make a hole in the system and ask whether we can get a better (or smaller) bound on the essential spectral radius. This problem is not simple and seems to have the same root as a prominent problem about the resonance of "chaotic scattering ". In fact, the real problem happens when the set of points that remain in the system forever has Hausdorff dimension larger than a " half " of the full dimension = 1 in the direction transversal to the flow. We present a results which improve " half " to " two thirds " under generic assumptions on the roof function.

S. Yamada: Construction of stationary solutions to the 4+1 Einstein equation with non-spherical blackholes

We introduce a new method for constructing a set of stationary solutions to the 4+1 dimensional vacuum Einstein equation with bi-axial symmetries. In 3+1 dimension, such solutions constitute the Kerr family of solutions with spherical horizon with two parameters; ADM mass and angular momentum. In 4+1 dimensions, however, there appear new possibilities of non-spherical event horizons, where the space-like section of the horizon is of the positive Yamabe type. In this collaborative work with Marcus Khuri and Gilbert Weinstein, we construct stationary solutions in the 4+1 dimension with non-spherical horizon, by solving an asymptotic Dirichlet problem where the target is a symmetric space SL(3)/SO(3) within the framework of the so-called Ernst reduction scheme.