Workshop:
Johnson homomorphisms and related topics
May 22-26, 2017

Lecture Hall in the Mathematical Science Building,
the University of Tokyo
3-8-1 Komaba Meguro-ku, Tokyo 153-8914, Japan

(ver.170420)

Time Table

May 22	**May 23**	**May 24**	**May 25**	**May 26**
10:00-11:00	10:00-11:00	10:00-11:00	10:00-11:00	10:00-11:00
Sakasai	Nozaki	Yasuda	Kordek	Vespa
11:20-12:20	11:20-12:20	11:20-12:20	11:20-12:20	11:20-12:20
Naef	Massuyeau	Conant	Nakai	Soulié
14:00-15:00	14:20-15:20		14:20-15:20	14:20-15:20
Tsuji	Habiro	excursion	Dèbes	Zeman
15:20-16:20	15:40-16:40		15:40-16:40	15:40-16:40
Nosaka	Morita		Hain	Randal-Williams
16:40-17:40	17:00-18:30		17:00-18:00	
Kobayashi	(TST) Hain		Berglund	
17:50-18:20	19:00-			
Omori	banquet			

Organizers: Nariya Kawazumi (Tokyo), Gwénaël Massuyeau (Strasbourg/CNRS), Hiroaki Nakamura (Osaka), Takuya Sakasai (Tokyo) and Christine Vespa (Strasbourg).

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Schedule with Titles

May 22 (Monday)

10:00–11:00,
Takuya Sakasai (University of Tokyo)
Johnson homomorphisms and symplectic representation theory

11:20–12:20,
Florian Naef (Université de Genève)
The Goldman-Turaev Lie bialgebra and its connection to the Kashiwara-Vergne problem

14:00–15:00,
Shunsuke Tsuji (University of Tokyo)
A formula for the action of Dehn twists on HOMFLY-PT type skein algebra and its application

15:20–16:20,
Takefumi Nosaka (Tokyo Institute of Technology)
Cocycles of nilpotent quotients of free groups; Massey products, Milnor invariants, and Johnson-Morita homomorphisms

16:40–17:40,
Ryoma Kobayashi (National Institute of Technology, Ishikawa College)
A normal generating set for the Torelli group of a compact non-orientable surface

17:50–18:20,
Genki Omori (Tokyo Institute of Technology)
Generators for the level 2 twist subgroup of the mapping class group of a non-orientable surface and its abelianization

May 23 (Tuesday)

10:00–11:00,
Yuta Nozaki (University of Tokyo)
An extension of the LMO functor

11:20–12:20,
Gwénaël Massuyeau (IRMA, Université de Strasbourg, CNRS)
The Kontsevich integral for bottom tangles in handlebodies: construction & properties

14:20–15:20,
Kazuo Habiro (RIMS, Kyoto University)
The Kontsevich integral for bottom tangles in handlebodies: algebraic aspects

15:40–16:40,
Shigeyuki Morita (University of Tokyo / Tokyo Institute of Technology (emeritus))
Torelli group versus invariants of homology spheres and beyond

17:00–18:30, Tuesday Seminar on Topology
Richard Hain (Duke University)
Johnson homomorphisms, stable and unstable

19:00–,
Banquet at the Seminar Room in the Komaba Faculty House

May 24 (Wednesday)

10:00–11:00,
Seidai Yasuda (Osaka University)
Linearized and derived double shuffle Lie algebras

11:20–12:20,
James Conant (University of Tennessee)
Quotients of the Lie Lie algebra

Informal Excursion

May 25 (Thursday)

10:00–11:00,
Kevin Kordek (Texas A&M University)
Picard groups of moduli spaces of curves with symmetry

11:20–12:20,
Hirofumi Nakai (Tokyo City University)
On the cohomology of moduli space of hyperelliptic curves

14:20–15:20,
Pierre Dèbes (Université Lille 1)
Topological Aspects in Inverse Galois Theory

15:40–16:40,
Richard Hain (Duke University)
Ihara Curves

17:00–18:00,
Alexander Berglund (Stockholm University)
Rational homotopy theory of automorphisms of simply connected manifolds

May 26 (Friday)

10:00–11:00,
Christine Vespa (IRMA, Université de Strasbourg)
Polynomial functors and the Johnson filtration

11:20–12:20,
Arthur Soulié (IRMA, Université de Strasbourg)
Action of the Long-Moody Construction on Polynomial Functors

14:20–15:20,
Tomáš Zeman (University of Oxford)
Homological stability for quotients of mapping class groups of surfaces

15:40–16:40,
Oscar Randal-Williams (University of Cambridge)
Stable twisted cohomology via scanning

Abstracts

Alexander Berglund (Stockholm University)

Rational homotopy theory of automorphisms of simply connected manifolds

We construct dg Lie algebra models, in the sense of Quillen's rational homotopy theory, for the homotopy automorphisms and the block diffeomorphisms of simply connected manifolds with boundary a sphere. The dg Lie algebras we obtain can be viewed as generalizations of the Lie algebras of symplectic derivations that appear in the study of homology of automorphisms of free groups and as targets of Johnson homomorphisms. In favorable situations, the models we construct can be used to compute the rational cohomology (in a stable range) of the classifying spaces in terms of certain decorated graph complexes. This is joint work with Ib Madsen.

James Conant (University of Tennessee)

Quotients of the Lie Lie algebra

We study two quotients of the Lie Lie algebra (the Lie algebra of symplectic derivations of the free Lie algebra), namely the abelianization and the the quotient by the Lie algebra generated by degree 1 elements. The abelianization has a very close connection to the homology of groups of automorphism groups of free groups, whereas the second is the so-called "Johnson cokernel," the cokernel of the Johnson homomorphism defined for mapping class groups of punctured surfaces

Pierre Dèbes (Université Lille 1)

Topological Aspects in Inverse Galois Theory

The still open Inverse Galois Problem consists in showing that every finite group is the Galois group of some field extension of Q. The modern geometric approach has arisen from the topological cover theory conjoined with the Riemann Existence Theorem. This has led to the Regular Inverse Galois Problem and the Hurwitz moduli space theory. We will review the key points of this approach which provides a full description of the solutions of the RIGP over C for a given group. We will then introduce a natural pre-order on this set of solutions, also topologically meaningful, and will present natural questions in this context, together with some answers.

Kazuo Habiro (RIMS, Kyoto University)

The Kontsevich integral for bottom tangles in handlebodies: algebraic aspects

I will focus on algebraic aspects of the functor from the category of bottom tangles in handlebodies to the category A of chord diagrams in handlebodies, which extends the usual Kontsevich integral. In particular, I plan to explain the presentation of the target category A, and the quasi-Hopf algebra structure in A. This is joint work with Gwénaël Massuyeau.

Richard Hain (Duke University) Tuesday Seminar on Topology

Johnson homomorphisms, stable and unstable

In this talk I will recall how motivic structures (Hodge and/or Galois) on the relative completions of mapping class groups yield non-trivial information about Johnson homomorphisms. I will explain how these motivic structures can be pasted, and why I believe that the genus 1 case is of fundamental importance in studying the higher genus (stable) case.

Richard Hain (Duke University)

Ihara Curves

Ihara and Nakamura (1998) generalized the Tate elliptic curve over Z[[q]] to higher genus curves. Such curves will be called "Ihara curves". In this talk I will describe motivic structures on the unipotent fundamental group of a smoothing of a nodal curve and prove that the Lie algebra of the unipotent fundamental group of an Ihara curve is a mixed Tate motive, unramified over the integers. One consequence of this result is that each pants decomposition of a surface S of type (g, n), where $2g - 2 + n > 0$, determines an action of the motivic Lie algebra $L(\sigma_3, \sigma_5, \sigma_7, \sigma_9, ...)$ on the unipotent fundamental group of S. I will discuss the implications of this result for conjectures of Morita as well as efforts, joint with Francis Brown, to describe this action.

Ryoma Kobayashi (National Institute of Technology, Ishikawa College)

A normal generating set for the Torelli group of a compact non-orientable surface

The mapping class group of a compact surface S is the group consisting of isotopy classes of all (orientation preserving) diffeomorphisms of S which fix each point of the boundary. The Torelli group of S is the subgroup of the mapping class group of S consisting of elements acting trivially on the integral first homology group of S. It is known that the Torelli group of a closed orientable surface is finitely generated. In this work, we constructed a normal generating set for the Torelli group of a compact non-orientable surface.

Kevin Kordek (Texas A&M University)

Picard groups of moduli spaces of curves with symmetry

In 1967, Mumford showed that the Picard group of the moduli space of genus g Riemann surfaces is isomorphic to the second integral cohomology of the genus g mapping class group. Techniques developed since that time have allowed one to productively study various generalizations of Mumford's original calculation. In this talk, I will explain how the theory of symmetric mapping class groups, developed by Birman-Hilden, Harvey, and others, can be used to understand - and sometimes exactly compute - the Picard groups of various moduli spaces of curves with symmetry, for example the moduli spaces of hyperelliptic curves. We will see, in particular, how the structure of the Johnson filtration is related to the problem of finite generation in certain cases.?

Gwénaël Massuyeau (IRMA, Université de Strasbourg, CNRS)

The Kontsevich integral for bottom tangles in handlebodies: construction & properties

In a joint work with Kazuo Habiro, we have extended the Kontsevich integral to a functor from the category of "bottom tangles in handlebodies" to a category of "Jacobi diagrams in handlebodies". In this talk, I will explain the construction of this functor and I will present some of its properties, including the universality among Vassiliev invariants and a close relationship with the TQFT derived from the Le-Murakami-Ohtsuki invariant.

Shigeyuki Morita (University of Tokyo / Tokyo Institute of Technology (emeritus))

Torelli group versus invariants of homology spheres and beyond

In the late 1980's, we proved that the Casson invariant appears in the difference between two filtrations of the Torelli group: the lower central series and the Johnson filtration. Then Hain obtained a deep result of giving infinitesimal presentation of the Torelli group. However, the problem of identifying the above difference remained open. In this talk, we first discuss whether Ohtsuki invariants of homology spheres appear there or not. We relate this problem to the determination of the second homology group of the Johnson image, which is a Lie subalgebra of the symplectic derivation Lie algebra of the free Lie algebra. Then we report our computation of it in low weights. Finally we mention our project of enhancing this picture in a broader context of homology cylinders where we consider the second homology group of the whole symplectic derivation Lie algebra. This talk is based on a joint work with Takuya Sakasai and Masaaki Suzuki.

Florian Naef (Université de Genève)

The Goldman-Turaev Lie bialgebra and its connection to the Kashiwara-Vergne problem

Using the intersection and self-intersection of loops on a surface one can define the Goldman-Turaev Lie bialgebra, and its non-commutative double avatar. On a genus zero surface with three boundary components the linearization problem of this structure is equivalent to the Kashiwara-Vergne problem. Motivated by this result a generalization of the Kashiwara-Vergne problem in higher genera is proposed and solutions are constructed in analogy with elliptic associators.

This is joint work with A. Alekseev, N. Kawazumi and Y. Kuno.

Hirofumi Nakai (Tokyo City University)

On the cohomology of moduli space of hyperelliptic curves

In the end of 20th-century Mike Hopkins and his coworkers established the theory of topological moduler forms, which is considered as the "universal" elliptic cohomology theory. In the computations of the homotopy groups of the representing spectrum, they distilled a Hopf algebroid from the Weierstrass equation and computed an Adams-Novikov type spectral sequence. In this talk we apply the similar method to the defining equation of hyperelliptic curves, and show the relavant topics to the mapping class groups.

Takefumi Nosaka (Tokyo Institute of Technology)

Cocycles of nilpotent quotients of free groups; Massey products, Milnor invariants, and Johnson-Morita homomorphisms

We focus on the cohomology of the k-th nilpotent quotient of the free group, F/F_k. In this talk, I describe all the group 2-, 3 -cocycles using Massey products, and give expressions of some 3-cocycles. As a corollary, I roughly explain the usefulness of the cocycles, and introduce simplified proofs of some results on Milnor invariant and the Johnson-Morita homomorphism.

Yuta Nozaki (University of Tokyo)

An extension of the LMO functor

Cheptea, Habiro and Massuyeau constructed the LMO functor, which is defined on a certain category of cobordisms between two surfaces with at most one boundary component. In this talk, we will extend the LMO functor to the case of any number of boundary components and show that our functor reflects a relation among the parts corresponding to the genera and boundary components of surfaces.

Genki Omori (Tokyo Institute of Technology)

Generators for the level 2 twist subgroup of the mapping class group of a non-orientable surface and its abelianization

Hirose and Sato gave a minimal generating set for the level 2 mapping class group of a closed non-orientable suface, and compute its abelianization. The twist subgroup of the mapping class group is the subgroup of the mapping class group generated by all Dehn twists. Then we consider the intersection of the level 2 mapping class group and the twist subgroup for a closed non-orientable surface, and call that the level 2 twist subgroup. In this talk, we give a finite generating set for the level 2 twist subgroup. By using the generating set, we also give the abelianization of the level 2 twist subgroup.

Oscar Randal-Williams (University of Cambridge)

Stable twisted cohomology via scanning

The technique of scanning, or the parameterised Pontrjagin–Thom construction, has been extraordinarily successful in calculating the cohomology of configuration spaces (McDuff), moduli spaces of Riemann surfaces (Madsen, Tillmann, Weiss), moduli spaces of graphs (Galatius), and moduli spaces of manifolds of higher dimension (Galatius, R-W, Botvinnik, Perlmutter), with constant coefficients. In each case the method also works to study the cohomology of moduli spaces of objects equipped with a "tangential structure". I will explain how choosing an auxiliary highly-symmetric tangential structure often lets one calculate the cohomology of these moduli spaces with large families of twisted coefficients, by exploiting the symmetries of the tangential structure and using a little representation theory.

Takuya Sakasai (University of Tokyo)

Johnson homomorphisms and symplectic representation theory

Abstract: We first review the definition of Johnson homomorphisms with their relationship to symplectic representation theory. Then we discuss a topological approach to understand certain deeper structures related to the Mumford-Morita-Miller (MMM) classes through Johnson homomorphisms. This is a joint work with Shigeyuki Morita and Masaaki Suzuki.

Arthur Soulié (IRMA, Université de Strasbourg)

Action of the Long-Moody Construction on Polynomial Functors

In 2016, Randal-Williams and Wahl proved homological stability with certain twisted coefficients for dif- ferent families of groups, in particular the one of braid groups. In fact, they obtain the stability for coefficients given by functors satisfying polynomial conditions. We only know few examples of such functors. Among them, we have the functor given by the unreduced Burau representations. In 1994, Long and Moody gave a construction on representations of braid groups which associates a representation of B_n with a representation of B_{n+1}. This construction complexifies in a sense the initial representation: for instance, starting from a dimension one representation, one obtains the unreduced Burau representation. In this talk, I will present this construction from a functorial point of view. I will explain that the construction of Long and Moody defines an endofunctor, called the Long-Moody functor, between a suitable category of functors. Then, after defining strong polynomial functors in this context, I will prove that the Long-Moody functor increases by one the degree of strong polynomiality of a strong polynomial functor. Thus, the Long-Moody construction will provide new examples of twisted coefficients entering in the framework of Randal-Williams and Wahl.

Shunsuke Tsuji (University of Tokyo)

A formula for the action of Dehn twists on HOMFLY-PT type skein algebra and its application

We define some filtrations of the HOMFLY-PT type skein modules and the HOMFLY-PT type skein algebra on an oriented surface, and define the completed HOMFLY-PT type skein modules and the completed HOMFLY-PT type skein algebra of the surface with respect to these filtrations. We give an explicit formula for the action of the Dehn twists on this completed skein modules in terms of the action of this completed skein algebra of the surface. As an application, using this formula, we construct an invariant for an integral homology 3-sphere which is an element of $\mathbb{Q}[\rho][[h]]$.

Christine Vespa (IRMA, Université de Strasbourg)

Polynomial functors and the Johnson filtration

The definition of polynomial functor in the sense of Eilenberg and Mac Lane can easily be extended to functors on a monoidal category whose unit is a null object. However several examples of functors having polynomial properties does not enter in this setting. In this talk I will give two notions of polynomial functors on a monoidal category where the unit is an initial object: the strong polynomial functors and the weak polynomial functors. Considering the IA-automorphisms groups of free groups, I will explain how the quotients of the lower central series and of the Johnson filtration give rive to polynomial functors. (This is a joint work with Aurélien Djament)

Seidai Yasuda (Osaka University)

Linearized and derived double shuffle Lie algebras

In this talk we consider the linearized double shuffle Lie algebra L for a commutative affine group scheme G originally introduced by Goncharov. Some conjecture on the structure of L is proposed by Goncharov and Brown when G is the additive group. Our first main result is that, when G is the multiplicative group, the Lie algebra L is freely generated by the depth one subspace. We also explain its consequences. Then we introduce the notion of derived double shuffle spaces and propose a conjectural description of the Goncharov's modular complexes.

Tomáš Zeman (University of Oxford)

Homological stability for quotients of mapping class groups of surfaces

It is classically known that both the mapping class groups of orientable surfaces and the symplectic groups over the integers (which can be thought of as quotients of the former by the Torelli subgroups) satisfy homological stability as the genus of the surface goes to infinity. Quotients of the mapping class group by subgroups in the Johnson filtration in some sense interpolate between these two sequences of groups, and I will talk about a recent proof that for every level of the Johnson filtration, the homology of these quotients also stabilises with the genus. Unlike many homological stability results in recent literature, this proof is not based on Quillen's method of finding highly connected complexes on which the groups act. A drawback of our argument, which makes use of the machinery of FI-modules, is that it gives no information on the range of stability.