Risa/Asir

```
[0] A=mat([3,0,-3,1,7],[1,1,1,-2,-1],[1,2,3,-4,-4],[0,-1,-2,-1,0]);
[ 3 0 -3 1 7 ]
[ 1 1 1 -2 -1 ]
[ 1 2 3 -4 -4 ]
[ 0 -1 -2 -1 0 ]
[1] os_md.mtoupper(A,0|opt=3,step=1,dviout=1)$
```

Then we have the following.

$$
\begin{aligned}
&\begin{pmatrix} 3 & 0 & -3 & 1 & 7 \\ 1 & 1 & 1 & -2 & -1 \\ 1 & 2 & 3 & -4 & -4 \\ 0 & -1 & -2 & -1 & 0 \end{pmatrix} \\
&\xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 1 & 1 & -2 & -1 \\ 3 & 0 & -3 & 1 & 7 \\ 1 & 2 & 3 & -4 & -4 \\ 0 & -1 & -2 & -1 & 0 \end{pmatrix} \\
&\xrightarrow{\text{line2} \ -= \ \text{line1}\times(3)} \begin{pmatrix} 1 & 1 & 1 & -2 & -1 \\ 0 & -3 & -6 & 7 & 10 \\ 1 & 2 & 3 & -4 & -4 \\ 0 & -1 & -2 & -1 & 0 \end{pmatrix} \\
&\xrightarrow{\text{line3} \ -= \ \text{line1}} \begin{pmatrix} 1 & 1 & 1 & -2 & -1 \\ 0 & -3 & -6 & 7 & 10 \\ 0 & 1 & 2 & -2 & -3 \\ 0 & -1 & -2 & -1 & 0 \end{pmatrix} \\
&\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 1 & 1 & -2 & -1 \\ 0 & 1 & 2 & -2 & -3 \\ 0 & -3 & -6 & 7 & 10 \\ 0 & -1 & -2 & -1 & 0 \end{pmatrix} \\
&\xrightarrow{\text{line1} \ -= \ \text{line2}} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 2 & -2 & -3 \\ 0 & -3 & -6 & 7 & 10 \\ 0 & -1 & -2 & -1 & 0 \end{pmatrix} \\
&\xrightarrow{\text{line3} \ += \ \text{line2}\times(3)} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 2 & -2 & -3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & -1 & -2 & -1 & 0 \end{pmatrix} \\
&\xrightarrow{\text{line4} \ += \ \text{line2}} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 2 & -2 & -3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 & -3 \end{pmatrix} \\
&\xrightarrow{\text{line2} \ += \ \text{line3}\times(2)} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -3 & -3 \end{pmatrix} \\
&\xrightarrow{\text{line4} \ += \ \text{line3}\times(3)} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}
\end{aligned}
\tag{1}
$$

The command

```
[2] os_md.mtoupper(A,-4|opt=3,step=1,dviout=1);
[ 1 0 -1 0 2 0 2 -1 0 ]
[ 0 1 2 0 -1 2 -13 7 0 ]
[ 0 0 0 1 1 1 -6 3 0 ]
[ 0 0 0 0 0 3 -19 10 1 ]
```

gives the following.

$$
\begin{aligned}
&\begin{pmatrix} 3 & 0 & -3 & 1 & 7 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & -2 & -1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & -4 & -4 & 0 & 0 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\
&\xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 1 & 1 & 1 & -2 & -1 & 0 & 1 & 0 & 0 \\ 3 & 0 & -3 & 1 & 7 & 1 & 0 & 0 & 0 \\ 1 & 2 & 3 & -4 & -4 & 0 & 0 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\
&\xrightarrow{\text{line2} \ -= \ \text{line1}\times(3)} \begin{pmatrix} 1 & 1 & 1 & -2 & -1 & 0 & 1 & 0 & 0 \\ 0 & -3 & -6 & 7 & 10 & 1 & -3 & 0 & 0 \\ 1 & 2 & 3 & -4 & -4 & 0 & 0 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\
&\xrightarrow{\text{line3} \ -= \ \text{line1}} \begin{pmatrix} 1 & 1 & 1 & -2 & -1 & 0 & 1 & 0 & 0 \\ 0 & -3 & -6 & 7 & 10 & 1 & -3 & 0 & 0 \\ 0 & 1 & 2 & -2 & -3 & 0 & -1 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\
&\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 1 & 1 & 1 & -2 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & -3 & 0 & -1 & 1 & 0 \\ 0 & -3 & -6 & 7 & 10 & 1 & -3 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\
&\xrightarrow{\text{line1} \ -= \ \text{line2}} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 & 0 & 2 & -1 & 0 \\ 0 & 1 & 2 & -2 & -3 & 0 & -1 & 1 & 0 \\ 0 & -3 & -6 & 7 & 10 & 1 & -3 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\
&\xrightarrow{\text{line3} \ += \ \text{line2}\times(3)} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 & 0 & 2 & -1 & 0 \\ 0 & 1 & 2 & -2 & -3 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -6 & 3 & 0 \\ 0 & -1 & -2 & -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \\
&\xrightarrow{\text{line4} \ += \ \text{line2}} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 & 0 & 2 & -1 & 0 \\ 0 & 1 & 2 & -2 & -3 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -6 & 3 & 0 \\ 0 & 0 & 0 & -3 & -3 & 0 & -1 & 1 & 1 \end{pmatrix} \\
&\xrightarrow{\text{line2} \ += \ \text{line3}\times(2)} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 & 0 & 2 & -1 & 0 \\ 0 & 1 & 2 & 0 & -1 & 2 & -13 & 7 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -6 & 3 & 0 \\ 0 & 0 & 0 & -3 & -3 & 0 & -1 & 1 & 1 \end{pmatrix} \\
&\xrightarrow{\text{line4} \ += \ \text{line3}\times(3)} \begin{pmatrix} 1 & 0 & -1 & 0 & 2 & 0 & 2 & -1 & 0 \\ 0 & 1 & 2 & 0 & -1 & 2 & -13 & 7 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -6 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -19 & 10 & 1 \end{pmatrix}
\end{aligned}
\tag{2}
$$

```
[3] os_md.mtoupper(os_md.s2m("32,53"),-2|opt=4,step=1,dviout=1)$
```

$$\begin{pmatrix}3 & 2 & 1 & 0\\5 & 3 & 0 & 1\end{pmatrix}$$
$$\xrightarrow{\text{line2 } -= \text{ line1}\times(2)} \begin{pmatrix}3 & 2 & 1 & 0\\-1 & -1 & -2 & 1\end{pmatrix}$$
$$\xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix}-1 & -1 & -2 & 1\\3 & 2 & 1 & 0\end{pmatrix}$$
(3)
$$\xrightarrow{\text{line1 } \times= \ (-1)} \begin{pmatrix}1 & 1 & 2 & -1\\3 & 2 & 1 & 0\end{pmatrix}$$
$$\xrightarrow{\text{line2 } -= \text{ line1}\times(3)} \begin{pmatrix}1 & 1 & 2 & -1\\0 & -1 & -5 & 3\end{pmatrix}$$
$$\xrightarrow{\text{line2 } \times= \ (-1)} \begin{pmatrix}1 & 1 & 2 & -1\\0 & 1 & 5 & -3\end{pmatrix}$$
$$\xrightarrow{\text{line1 } -= \text{ line2}} \begin{pmatrix}1 & 0 & -3 & 2\\0 & 1 & 5 & -3\end{pmatrix}$$

```
[4] os_md.mtoupper(mat([a,2],[a^2+a+2,6*a+2]),0|opt=5,step=1,dviout=1)$
```

$$\begin{pmatrix}a & 2\\a^2+a+2 & 2(3a+1)\end{pmatrix}$$
$$\xrightarrow{\text{line2 } += \text{ line1}\times(-(a+1))} \begin{pmatrix}a & 2\\2 & 4a\end{pmatrix}$$
$$\xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix}2 & 4a\\a & 2\end{pmatrix}$$
$$\xrightarrow{\text{line1 } \times= \ \left(\frac{1}{2}\right)} \begin{pmatrix}1 & 2a\\a & 2\end{pmatrix}$$
$$\xrightarrow{\text{line2 } += \text{ line1}\times(-a)} \begin{pmatrix}1 & 2a\\0 & -2(a-1)(a+1)\end{pmatrix}$$

(4) If $a=1$,
$$\begin{pmatrix}1 & 2\\0 & 0\end{pmatrix}$$
If $a=-1$,
$$\begin{pmatrix}1 & -2\\0 & 0\end{pmatrix}$$
If $(a-1)(a+1)\neq 0$,
$$\xrightarrow{\text{line2 } \times= \ \left(\frac{-1}{2(a-1)(a+1)}\right)} \begin{pmatrix}1 & 2a\\0 & 1\end{pmatrix}$$
$$\xrightarrow{\text{line1 } += \text{ line2}\times(-2a)} \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

```
[5]  os_md.mtoupper(mat([a,b],[c,d]),-2|opt=6,step=1,dviout=1)$
```

$$\begin{pmatrix} a & b & 1 & 0 \\ c & d & 0 & 1 \end{pmatrix}$$

If $a = 0$,

$$\begin{pmatrix} 0 & b & 1 & 0 \\ c & d & 0 & 1 \end{pmatrix}$$

If $c = 0$,

$$\begin{pmatrix} 0 & b & 1 & 0 \\ 0 & d & 0 & 1 \end{pmatrix}$$

If $b = 0$,

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & d & 0 & 1 \end{pmatrix}$$

If $d = 0$,

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

If $d \neq 0$,

$$\xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 0 & d & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \times= \left(\frac{1}{d}\right)} \begin{pmatrix} 0 & 1 & 0 & \frac{1}{d} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If $b \neq 0$,

$$\xrightarrow{\text{line1} \times= \left(\frac{1}{b}\right)} \begin{pmatrix} 0 & 1 & \frac{1}{b} & 0 \\ 0 & d & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} += \text{line1}\times(-d)} \begin{pmatrix} 0 & 1 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{-d}{b} & 1 \end{pmatrix}$$

If $c \neq 0$,

(5)

$$\xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} c & d & 0 & 1 \\ 0 & b & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \times= \left(\frac{1}{c}\right)} \begin{pmatrix} 1 & \frac{d}{c} & 0 & \frac{1}{c} \\ 0 & b & 1 & 0 \end{pmatrix}$$

If $b = 0$,

$$\begin{pmatrix} 1 & \frac{d}{c} & 0 & \frac{1}{c} \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

If $b \neq 0$,

$$\xrightarrow{\text{line2} \times= \left(\frac{1}{b}\right)} \begin{pmatrix} 1 & \frac{d}{c} & 0 & \frac{1}{c} \\ 0 & 1 & \frac{1}{b} & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1} += \text{line2}\times\left(\frac{-d}{c}\right)} \begin{pmatrix} 1 & 0 & \frac{-d}{cb} & \frac{1}{c} \\ 0 & 1 & \frac{1}{b} & 0 \end{pmatrix}$$

If $a \neq 0$,

$$\xrightarrow{\text{line1} \times= \left(\frac{1}{a}\right)} \begin{pmatrix} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ c & d & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} += \text{line1}\times(-c)} \begin{pmatrix} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & \frac{da-cb}{a} & \frac{-c}{a} & 1 \end{pmatrix}$$

If $d = \dfrac{cb}{a}$,

$$\begin{pmatrix} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & 0 & \frac{-c}{a} & 1 \end{pmatrix}$$

If $da - cb \neq 0$,

$$\xrightarrow{\text{line2} \times= \left(\frac{a}{da-cb}\right)} \begin{pmatrix} 1 & \frac{b}{a} & \frac{1}{a} & 0 \\ 0 & 1 & \frac{-c}{da-cb} & \frac{a}{da-cb} \end{pmatrix}$$

$$\xrightarrow{\text{line1} += \text{line2}\times\left(\frac{-b}{a}\right)} \begin{pmatrix} 1 & 0 & \frac{d}{da-cb} & \frac{-b}{da-cb} \\ 0 & 1 & \frac{-c}{da-cb} & \frac{a}{da-cb} \end{pmatrix}$$

```
[6] os_md.mtoupper(mat([a^4-b^2,b]),0|step=1,opt=6,dviout=1)$
```

(6)

$$\begin{pmatrix}(a^2-b)(a^2+b) & b\end{pmatrix}$$

If $b=a^2$,

$$\begin{pmatrix}0 & a^2\end{pmatrix}$$

If $a=0$,

$$\begin{pmatrix}0 & 0\end{pmatrix}$$

If $a\neq 0$,

$$\xrightarrow{\text{line1 }\times=\left(\frac{1}{a^2}\right)}\begin{pmatrix}0 & 1\end{pmatrix}$$

If $b=-a^2$,

$$\begin{pmatrix}0 & -a^2\end{pmatrix}$$

If $a=0$,

$$\begin{pmatrix}0 & 0\end{pmatrix}$$

If $a\neq 0$,

$$\xrightarrow{\text{line1 }\times=\left(\frac{-1}{a^2}\right)}\begin{pmatrix}0 & 1\end{pmatrix}$$

If $(a^2-b)(a^2+b)\neq 0$,

$$\xrightarrow{\text{line1 }\times=\left(\frac{1}{(a^2-b)(a^2+b)}\right)}\begin{pmatrix}1 & \frac{b}{(a^2-b)(a^2+b)}\end{pmatrix}$$

```
[7] os_md.mtoupper(mat([a^4+b^2,b]),0|step=1,opt=6,dviout=1)$
```

(7)

$$\begin{pmatrix}a^4+b^2 & b\end{pmatrix}$$

Assume $a^4+b^2\neq 0$,

$$\xrightarrow{\text{line1 }\times=\left(\frac{1}{a^4+b^2}\right)}\begin{pmatrix}1 & \frac{b}{a^4+b^2}\end{pmatrix}$$

```
[8] os_md.mtoupper(mat([a^4+a^2*b^2]),0|step=1,opt=6,dviout=1)$
```

(8)

$$\begin{pmatrix}a^2(a^2+b^2) & b\end{pmatrix}$$

If $a=0$,

$$\begin{pmatrix}0 & b\end{pmatrix}$$

If $b=0$,

$$\begin{pmatrix}0 & 0\end{pmatrix}$$

If $b\neq 0$,

$$\xrightarrow{\text{line1 }\times=\left(\frac{1}{b}\right)}\begin{pmatrix}0 & 1\end{pmatrix}$$

Assume $a(a^2+b^2)\neq 0$,

$$\xrightarrow{\text{line1 }\times=\left(\frac{1}{a^2(a^2+b^2)}\right)}\begin{pmatrix}1 & \frac{b}{a^2(a^2+b^2)}\end{pmatrix}$$

```
[9] os_md.mtoupper(mat([a,b,c],[d,e,f],[p,q,r]),0|step=1,opt=6,dviout=1,tab=4)$
```

$$\begin{pmatrix} a & b & c & 1 & 0 & 0 \\ d & e & f & 0 & 1 & 0 \\ p & q & r & 0 & 0 & 1 \end{pmatrix}$$

If $a = 0$,

$$\begin{pmatrix} 0 & b & c & 1 & 0 & 0 \\ d & e & f & 0 & 1 & 0 \\ p & q & r & 0 & 0 & 1 \end{pmatrix}$$

If $d = 0$,

$$\begin{pmatrix} 0 & b & c & 1 & 0 & 0 \\ 0 & e & f & 0 & 1 & 0 \\ p & q & r & 0 & 0 & 1 \end{pmatrix}$$

If $p = 0$,

$$\begin{pmatrix} 0 & b & c & 1 & 0 & 0 \\ 0 & e & f & 0 & 1 & 0 \\ 0 & q & r & 0 & 0 & 1 \end{pmatrix}$$

If $b = 0$,

$$\begin{pmatrix} 0 & 0 & c & 1 & 0 & 0 \\ 0 & e & f & 0 & 1 & 0 \\ 0 & q & r & 0 & 0 & 1 \end{pmatrix}$$

If $e = 0$,

$$\begin{pmatrix} 0 & 0 & c & 1 & 0 & 0 \\ 0 & 0 & f & 0 & 1 & 0 \\ 0 & q & r & 0 & 0 & 1 \end{pmatrix}$$

If $q = 0$,

$$\begin{pmatrix} 0 & 0 & c & 1 & 0 & 0 \\ 0 & 0 & f & 0 & 1 & 0 \\ 0 & 0 & r & 0 & 0 & 1 \end{pmatrix}$$

If $c = 0$,

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & f & 0 & 1 & 0 \\ 0 & 0 & r & 0 & 0 & 1 \end{pmatrix}$$

If $f = 0$,

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & r & 0 & 0 & 1 \end{pmatrix}$$

If $r = 0$,

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

If $r \neq 0$,

$$\xrightarrow{\text{line1} \leftrightarrow \text{line3}} \begin{pmatrix} 0 & 0 & r & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \ \times= \ \left(\frac{1}{r}\right)} \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & \frac{1}{r} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

If $f \neq 0$,

$$\xrightarrow{\text{line1} \leftrightarrow \text{line2}} \begin{pmatrix} 0 & 0 & f & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1} \ \times= \ \left(\frac{1}{f}\right)} \begin{pmatrix} 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3} \ += \ \text{line1}\times(-r)} \begin{pmatrix} 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-r}{f} & 1 \end{pmatrix}$$

If $c \neq 0$,

$$\xrightarrow{\text{line1 } \times= \left(\frac{1}{c}\right)} \begin{pmatrix} 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & f & 0 & 1 & 0 \\ 0 & 0 & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line1} \times (-f)} \begin{pmatrix} 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & 0 & \frac{-f}{c} & 1 & 0 \\ 0 & 0 & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line1} \times (-r)} \begin{pmatrix} 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & 0 & \frac{-f}{c} & 1 & 0 \\ 0 & 0 & 0 & \frac{-r}{c} & 0 & 1 \end{pmatrix}$$

If $q \neq 0$,

$$\xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} 0 & q & r & 0 & 0 & 1 \\ 0 & 0 & f & 0 & 1 & 0 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } \times= \left(\frac{1}{q}\right)} \begin{pmatrix} 0 & 1 & \frac{r}{q} & 0 & 0 & \frac{1}{q} \\ 0 & 0 & f & 0 & 1 & 0 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

If $f = 0$,

$$\begin{pmatrix} 0 & 1 & \frac{r}{q} & 0 & 0 & \frac{1}{q} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

If $c = 0$,

$$\begin{pmatrix} 0 & 1 & \frac{r}{q} & 0 & 0 & \frac{1}{q} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

If $c \neq 0$,

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 0 & 1 & \frac{r}{q} & 0 & 0 & \frac{1}{q} \\ 0 & 0 & c & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{c}\right)} \begin{pmatrix} 0 & 1 & \frac{r}{q} & 0 & 0 & \frac{1}{q} \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2} \times \left(\frac{-r}{q}\right)} \begin{pmatrix} 0 & 1 & 0 & \frac{-r}{cq} & 0 & \frac{1}{q} \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

If $f \neq 0$,

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{f}\right)} \begin{pmatrix} 0 & 1 & \frac{r}{q} & 0 & 0 & \frac{1}{q} \\ 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2} \times \left(\frac{-r}{q}\right)} \begin{pmatrix} 0 & 1 & 0 & 0 & \frac{-r}{fq} & \frac{1}{q} \\ 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2} \times (-c)} \begin{pmatrix} 0 & 1 & 0 & 0 & \frac{-r}{fq} & \frac{1}{q} \\ 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \\ 0 & 0 & 0 & 1 & \frac{-c}{f} & 0 \end{pmatrix}$$

If $e \neq 0$,

$$\xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix} 0 & e & f & 0 & 1 & 0 \\ 0 & 0 & c & 1 & 0 & 0 \\ 0 & q & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } \times= \left(\frac{1}{e}\right)} \begin{pmatrix} 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & c & 1 & 0 & 0 \\ 0 & q & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line1} \times (-q)} \begin{pmatrix} 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & c & 1 & 0 & 0 \\ 0 & 0 & \frac{-fq+er}{e} & 0 & \frac{-q}{e} & 1 \end{pmatrix}$$

If $c = 0$,

$$\begin{pmatrix} 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-fq+er}{e} & 0 & \frac{-q}{e} & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & \frac{-fq+er}{e} & 0 & \frac{-q}{e} & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times = \left(\frac{-e}{fq-er}\right)} \begin{pmatrix} 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & 1 & 0 & \frac{q}{fq-er} & \frac{-e}{fq-er} \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-f}{e}\right)}$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 & \frac{-r}{fq-er} & \frac{f}{fq-er} \\ 0 & 0 & 1 & 0 & \frac{q}{fq-er} & \frac{-e}{fq-er} \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

If $c \neq 0$,

$$\xrightarrow{\text{line2 } \times = \left(\frac{1}{c}\right)} \begin{pmatrix} 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & \frac{-fq+er}{e} & 0 & \frac{-q}{e} & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-f}{e}\right)} \begin{pmatrix} 0 & 1 & 0 & \frac{-f}{ec} & \frac{1}{e} & 0 \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & \frac{-fq+er}{e} & 0 & \frac{-q}{e} & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2}\times\left(\frac{fq-er}{e}\right)} \begin{pmatrix} 0 & 1 & 0 & \frac{-f}{ec} & \frac{1}{e} & 0 \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & 0 & \frac{fq-er}{ec} & \frac{-q}{e} & 1 \end{pmatrix}$$

If $b \neq 0$,

$$\xrightarrow{\text{line1 } \times = \left(\frac{1}{b}\right)} \begin{pmatrix} 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & e & f & 0 & 1 & 0 \\ 0 & q & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line1}\times(-e)} \begin{pmatrix} 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & \frac{fb-ec}{b} & \frac{-e}{b} & 1 & 0 \\ 0 & q & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line1}\times(-q)} \begin{pmatrix} 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & \frac{fb-ec}{b} & \frac{-e}{b} & 1 & 0 \\ 0 & 0 & \frac{-cq+br}{b} & \frac{-q}{b} & 0 & 1 \end{pmatrix}$$

If $f = \dfrac{ec}{b}$,

$$\begin{pmatrix} 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & 0 & \frac{-e}{b} & 1 & 0 \\ 0 & 0 & \frac{-cq+br}{b} & \frac{-q}{b} & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2} \leftrightarrow \text{line3}} \begin{pmatrix} 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & \frac{-cq+br}{b} & \frac{-q}{b} & 0 & 1 \\ 0 & 0 & 0 & \frac{-e}{b} & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times = \left(\frac{-b}{cq-br}\right)} \begin{pmatrix} 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & 1 & \frac{q}{cq-br} & 0 & \frac{-b}{cq-br} \\ 0 & 0 & 0 & \frac{-e}{b} & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-c}{b}\right)} \begin{pmatrix} 0 & 1 & 0 & \frac{-r}{cq-br} & 0 & \frac{c}{cq-br} \\ 0 & 0 & 1 & \frac{q}{cq-br} & 0 & \frac{-b}{cq-br} \\ 0 & 0 & 0 & \frac{-e}{b} & 1 & 0 \end{pmatrix}$$

If $fb - ec \neq 0$,

$$\xrightarrow{\text{line2 } \times = \left(\frac{b}{fb-ec}\right)} \begin{pmatrix} 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & 1 & \frac{-e}{fb-ec} & \frac{b}{fb-ec} & 0 \\ 0 & 0 & \frac{-cq+br}{b} & \frac{-q}{b} & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-c}{b}\right)}$$

$$\begin{pmatrix} 0 & 1 & 0 & \frac{f}{fb-ec} & \frac{-c}{fb-ec} & 0 \\ 0 & 0 & 1 & \frac{-e}{fb-ec} & \frac{b}{fb-ec} & 0 \\ 0 & 0 & \frac{-cq+br}{b} & \frac{-q}{b} & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2}\times\left(\frac{cq-br}{b}\right)}$$

$$\begin{pmatrix} 0 & 1 & 0 & \frac{f}{fb-ec} & \frac{-c}{fb-ec} & 0 \\ 0 & 0 & 1 & \frac{-e}{fb-ec} & \frac{b}{fb-ec} & 0 \\ 0 & 0 & 0 & \frac{-fq+er}{fb-ec} & \frac{cq-br}{fb-ec} & 1 \end{pmatrix}$$

If $p \neq 0$,

$$\xrightarrow{\text{line1 } \leftrightarrow \text{ line3}} \begin{pmatrix} p & q & r & 0 & 0 & 1 \\ 0 & e & f & 0 & 1 & 0 \\ 0 & b & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } \times= \left(\frac{1}{p}\right)} \begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & e & f & 0 & 1 & 0 \\ 0 & b & c & 1 & 0 & 0 \end{pmatrix}$$

If $e = 0$,

$$\begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & 0 & f & 0 & 1 & 0 \\ 0 & b & c & 1 & 0 & 0 \end{pmatrix}$$

If $b = 0$,

$$\begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & 0 & f & 0 & 1 & 0 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

If $f = 0$,

$$\begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

If $c = 0$,

$$\begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

If $c \neq 0$,

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & 0 & c & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{c}\right)} \begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-r}{p}\right)} \begin{pmatrix} 1 & \frac{q}{p} & 0 & \frac{-r}{cp} & 0 & \frac{1}{p} \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

If $f \neq 0$,

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{f}\right)} \begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-r}{p}\right)} \begin{pmatrix} 1 & \frac{q}{p} & 0 & 0 & \frac{-r}{fp} & \frac{1}{p} \\ 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2}\times(-c)} \begin{pmatrix} 1 & \frac{q}{p} & 0 & 0 & \frac{-r}{fp} & \frac{1}{p} \\ 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \\ 0 & 0 & 0 & 1 & \frac{-c}{f} & 0 \end{pmatrix}$$

If $b \neq 0$,

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & b & c & 1 & 0 & 0 \\ 0 & 0 & f & 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{b}\right)} \begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & f & 0 & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-q}{p}\right)} \begin{pmatrix} 1 & 0 & \frac{-cq+br}{bp} & \frac{-q}{bp} & 0 & \frac{1}{p} \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & f & 0 & 1 & 0 \end{pmatrix}$$

If $f = 0$,

$$\begin{pmatrix} 1 & 0 & \frac{-cq+br}{bp} & \frac{-q}{bp} & 0 & \frac{1}{p} \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

If $f \neq 0$,

$$\xrightarrow{\text{line3 } \times= \left(\frac{1}{f}\right)} \begin{pmatrix} 1 & 0 & \frac{-cq+br}{bp} & \frac{-q}{bp} & 0 & \frac{1}{p} \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line3}\times\left(\frac{cq-br}{bp}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{-q}{bp} & \frac{cq-br}{fbp} & \frac{1}{p} \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line3}\times\left(\frac{-c}{b}\right)} \begin{pmatrix} 1 & 0 & 0 & \frac{-q}{bp} & \frac{cq-br}{fbp} & \frac{1}{p} \\ 0 & 1 & 0 & \frac{1}{b} & \frac{-c}{fb} & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{f} & 0 \end{pmatrix}$$

If $e \neq 0$,

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{e}\right)} \begin{pmatrix} 1 & \frac{q}{p} & \frac{r}{p} & 0 & 0 & \frac{1}{p} \\ 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & b & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-q}{p}\right)} \begin{pmatrix} 1 & 0 & \frac{-fq+er}{ep} & 0 & \frac{-q}{ep} & \frac{1}{p} \\ 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & b & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2}\times(-b)} \begin{pmatrix} 1 & 0 & \frac{-fq+er}{ep} & 0 & \frac{-q}{ep} & \frac{1}{p} \\ 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & \frac{-fb+ec}{e} & 1 & \frac{-b}{e} & 0 \end{pmatrix}$$

If $c = \frac{fb}{e}$,

$$\begin{pmatrix} 1 & 0 & \frac{-fq+er}{ep} & 0 & \frac{-q}{ep} & \frac{1}{p} \\ 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & 0 & 1 & \frac{-b}{e} & 0 \end{pmatrix}$$

If $fb - ec \neq 0$,

$$\xrightarrow{\text{line3 } \times= \left(\frac{-e}{fb-ec}\right)} \begin{pmatrix} 1 & 0 & \frac{-fq+er}{ep} & 0 & \frac{-q}{ep} & \frac{1}{p} \\ 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & 1 & \frac{-e}{fb-ec} & \frac{b}{fb-ec} & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line3}\times\left(\frac{fq-er}{ep}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{-fq+er}{(fb-ec)p} & \frac{cq-br}{(fb-ec)p} & \frac{1}{p} \\ 0 & 1 & \frac{f}{e} & 0 & \frac{1}{e} & 0 \\ 0 & 0 & 1 & \frac{-e}{fb-ec} & \frac{b}{fb-ec} & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line3}\times\left(\frac{-f}{e}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{-fq+er}{(fb-ec)p} & \frac{cq-br}{(fb-ec)p} & \frac{1}{p} \\ 0 & 1 & 0 & \frac{f}{fb-ec} & \frac{-c}{fb-ec} & 0 \\ 0 & 0 & 1 & \frac{-e}{fb-ec} & \frac{b}{fb-ec} & 0 \end{pmatrix}$$

If $d \neq 0$,

$$\xrightarrow{\text{line1 } \leftrightarrow \text{ line2}} \begin{pmatrix} d & e & f & 0 & 1 & 0 \\ 0 & b & c & 1 & 0 & 0 \\ p & q & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } \times= \left(\frac{1}{d}\right)} \begin{pmatrix} 1 & \frac{e}{d} & \frac{f}{d} & 0 & \frac{1}{d} & 0 \\ 0 & b & c & 1 & 0 & 0 \\ p & q & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line1}\times(-p)} \begin{pmatrix} 1 & \frac{e}{d} & \frac{f}{d} & 0 & \frac{1}{d} & 0 \\ 0 & b & c & 1 & 0 & 0 \\ 0 & \frac{-ep+dq}{d} & \frac{-fp+dr}{d} & 0 & \frac{-p}{d} & 1 \end{pmatrix}$$

If $b = 0$,

$$\begin{pmatrix} 1 & \frac{e}{d} & \frac{f}{d} & 0 & \frac{1}{d} & 0 \\ 0 & 0 & c & 1 & 0 & 0 \\ 0 & \frac{-ep+dq}{d} & \frac{-fp+dr}{d} & 0 & \frac{-p}{d} & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & \frac{e}{d} & \frac{f}{d} & 0 & \frac{1}{d} & 0 \\ 0 & \frac{-ep+dq}{d} & \frac{-fp+dr}{d} & 0 & \frac{-p}{d} & 1 \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= \left(\frac{-d}{ep-dq}\right)} \begin{pmatrix} 1 & \frac{e}{d} & \frac{f}{d} & 0 & \frac{1}{d} & 0 \\ 0 & 1 & \frac{fp-dr}{ep-dq} & 0 & \frac{p}{ep-dq} & \frac{-d}{ep-dq} \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-e}{d}\right)}$$

$$\begin{pmatrix} 1 & 0 & \frac{-fq+er}{ep-dq} & 0 & \frac{-q}{ep-dq} & \frac{e}{ep-dq} \\ 0 & 1 & \frac{fp-dr}{ep-dq} & 0 & \frac{p}{ep-dq} & \frac{-d}{ep-dq} \\ 0 & 0 & c & 1 & 0 & 0 \end{pmatrix}$$

If $c = 0$,

$$\begin{pmatrix} 1 & 0 & \frac{-fq+er}{ep-dq} & 0 & \frac{-q}{ep-dq} & \frac{e}{ep-dq} \\ 0 & 1 & \frac{fp-dr}{ep-dq} & 0 & \frac{p}{ep-dq} & \frac{-d}{ep-dq} \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

If $c \neq 0$,

$$\xrightarrow{\text{line3 } \times= \left(\frac{1}{c}\right)} \begin{pmatrix} 1 & 0 & \frac{-fq+er}{ep-dq} & 0 & \frac{-q}{ep-dq} & \frac{e}{ep-dq} \\ 0 & 1 & \frac{fp-dr}{ep-dq} & 0 & \frac{p}{ep-dq} & \frac{-d}{ep-dq} \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line3}\times\left(\frac{fq-er}{ep-dq}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{fq-er}{ecp-dcq} & \frac{-q}{ep-dq} & \frac{e}{ep-dq} \\ 0 & 1 & \frac{fp-dr}{ep-dq} & 0 & \frac{p}{ep-dq} & \frac{-d}{ep-dq} \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line3}\times\left(\frac{-fp+dr}{ep-dq}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{fq-er}{ecp-dcq} & \frac{-q}{ep-dq} & \frac{e}{ep-dq} \\ 0 & 1 & 0 & \frac{-fp+dr}{ecp-dcq} & \frac{p}{ep-dq} & \frac{-d}{ep-dq} \\ 0 & 0 & 1 & \frac{1}{c} & 0 & 0 \end{pmatrix}$$

If $b \neq 0$,

$$\xrightarrow{\text{line2 } \times= \left(\frac{1}{b}\right)} \begin{pmatrix} 1 & \frac{e}{d} & \frac{f}{d} & 0 & \frac{1}{d} & 0 \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & \frac{-ep+dq}{d} & \frac{-fp+dr}{d} & 0 & \frac{-p}{d} & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2}\times\left(\frac{-e}{d}\right)} \begin{pmatrix} 1 & 0 & \frac{fb-ec}{db} & \frac{-e}{db} & \frac{1}{d} & 0 \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & \frac{-ep+dq}{d} & \frac{-fp+dr}{d} & 0 & \frac{-p}{d} & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2}\times\left(\frac{ep-dq}{d}\right)} \begin{pmatrix} 1 & 0 & \frac{fb-ec}{db} & \frac{-e}{db} & \frac{1}{d} & 0 \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & \frac{(-fb+ec)p-dcq+dbr}{db} & \frac{ep-dq}{db} & \frac{-p}{d} & 1 \end{pmatrix}$$

If $r = \dfrac{(fb-ec)p+dcq}{db}$,

$$\begin{pmatrix} 1 & 0 & \frac{fb-ec}{db} & \frac{-e}{db} & \frac{1}{d} & 0 \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & 0 & \frac{ep-dq}{db} & \frac{-p}{d} & 1 \end{pmatrix}$$

If $(fb-ec)p+dcq-dbr \neq 0$,

$$\xrightarrow{\text{line3 } \times= \left(\frac{-db}{(fb-ec)p+dcq-dbr}\right)}$$

$$\begin{pmatrix} 1 & 0 & \frac{fb-ec}{db} & \frac{-e}{db} & \frac{1}{d} & 0 \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & 1 & \frac{-ep+dq}{(fb-ec)p+dcq-dbr} & \frac{bp}{(fb-ec)p+dcq-dbr} & \frac{-db}{(fb-ec)p+dcq-dbr} \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{line3} \times \left(\frac{-fb+ec}{db}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{-fq+er}{(fb-ec)p+dcq-dbr} & \frac{cq-br}{(fb-ec)p+dcq-dbr} & \frac{fb-ec}{(fb-ec)p+dcq-dbr} \\ 0 & 1 & \frac{c}{b} & \frac{1}{b} & 0 & 0 \\ 0 & 0 & 1 & \frac{-ep+dq}{(fb-ec)p+dcq-dbr} & \frac{bp}{(fb-ec)p+dcq-dbr} & \frac{-db}{(fb-ec)p+dcq-dbr} \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{line3} \times \left(\frac{-c}{b}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{-fq+er}{(fb-ec)p+dcq-dbr} & \frac{cq-br}{(fb-ec)p+dcq-dbr} & \frac{fb-ec}{(fb-ec)p+dcq-dbr} \\ 0 & 1 & 0 & \frac{fp-dr}{(fb-ec)p+dcq-dbr} & \frac{-cp}{(fb-ec)p+dcq-dbr} & \frac{dc}{(fb-ec)p+dcq-dbr} \\ 0 & 0 & 1 & \frac{-ep+dq}{(fb-ec)p+dcq-dbr} & \frac{bp}{(fb-ec)p+dcq-dbr} & \frac{-db}{(fb-ec)p+dcq-dbr} \end{pmatrix}$$

If $a \neq 0$,

$$\xrightarrow{\text{line1 } \times= \left(\frac{1}{a}\right)} \begin{pmatrix} 1 & \frac{b}{a} & \frac{c}{a} & \frac{1}{a} & 0 & 0 \\ d & e & f & 0 & 1 & 0 \\ p & q & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{line1} \times (-d)} \begin{pmatrix} 1 & \frac{b}{a} & \frac{c}{a} & \frac{1}{a} & 0 & 0 \\ 0 & \frac{ea-db}{a} & \frac{fa-dc}{a} & \frac{-d}{a} & 1 & 0 \\ p & q & r & 0 & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{line1} \times (-p)} \begin{pmatrix} 1 & \frac{b}{a} & \frac{c}{a} & \frac{1}{a} & 0 & 0 \\ 0 & \frac{ea-db}{a} & \frac{fa-dc}{a} & \frac{-d}{a} & 1 & 0 \\ 0 & \frac{-bp+aq}{a} & \frac{-cp+ar}{a} & \frac{-p}{a} & 0 & 1 \end{pmatrix}$$

If $e = \dfrac{db}{a}$,

$$\begin{pmatrix} 1 & \frac{b}{a} & \frac{c}{a} & \frac{1}{a} & 0 & 0 \\ 0 & 0 & \frac{fa-dc}{a} & \frac{-d}{a} & 1 & 0 \\ 0 & \frac{-bp+aq}{a} & \frac{-cp+ar}{a} & \frac{-p}{a} & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \leftrightarrow \text{ line3}} \begin{pmatrix} 1 & \frac{b}{a} & \frac{c}{a} & \frac{1}{a} & 0 & 0 \\ 0 & \frac{-bp+aq}{a} & \frac{-cp+ar}{a} & \frac{-p}{a} & 0 & 1 \\ 0 & 0 & \frac{fa-dc}{a} & \frac{-d}{a} & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } \times= \left(\frac{-a}{bp-aq}\right)} \begin{pmatrix} 1 & \frac{b}{a} & \frac{c}{a} & \frac{1}{a} & 0 & 0 \\ 0 & 1 & \frac{cp-ar}{bp-aq} & \frac{p}{bp-aq} & 0 & \frac{-a}{bp-aq} \\ 0 & 0 & \frac{fa-dc}{a} & \frac{-d}{a} & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{line2} \times \left(\frac{-b}{a}\right)}$$

$$\begin{pmatrix} 1 & 0 & \frac{-cq+br}{bp-aq} & \frac{-q}{bp-aq} & 0 & \frac{b}{bp-aq} \\ 0 & 1 & \frac{cp-ar}{bp-aq} & \frac{p}{bp-aq} & 0 & \frac{-a}{bp-aq} \\ 0 & 0 & \frac{fa-dc}{a} & \frac{-d}{a} & 1 & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } \times= \left(\frac{a}{fa-dc}\right)}$$

$$\begin{pmatrix} 1 & 0 & \frac{-cq+br}{bp-aq} & \frac{-q}{bp-aq} & 0 & \frac{b}{bp-aq} \\ 0 & 1 & \frac{cp-ar}{bp-aq} & \frac{p}{bp-aq} & 0 & \frac{-a}{bp-aq} \\ 0 & 0 & 1 & \frac{-d}{fa-dc} & \frac{a}{fa-dc} & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{line3} \times \left(\frac{cq-br}{bp-aq}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{-faq+dbr}{(fba-dcb)p+(-fa^2+dca)q} & \frac{caq-bar}{(fba-dcb)p+(-fa^2+dca)q} & \frac{b}{bp-aq} \\ 0 & 1 & \frac{cp-ar}{bp-aq} & \frac{p}{bp-aq} & 0 & \frac{-a}{bp-aq} \\ 0 & 0 & 1 & \frac{-d}{fa-dc} & \frac{a}{fa-dc} & 0 \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{line3} \times \left(\frac{-cp+ar}{bp-aq}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{-faq+dbr}{(fba-dcb)p+(-fa^2+dca)q} & \frac{caq-bar}{(fba-dcb)p+(-fa^2+dca)q} & \frac{b}{bp-aq} \\ 0 & 1 & 0 & \frac{fap-dar}{(fba-dcb)p+(-fa^2+dca)q} & \frac{-cap+a^2r}{(fba-dcb)p+(-fa^2+dca)q} & \frac{-a}{bp-aq} \\ 0 & 0 & 1 & \frac{-d}{fa-dc} & \frac{a}{fa-dc} & 0 \end{pmatrix}$$

If $ea-db \neq 0,$

$$\xrightarrow{\text{line2 } \times= \left(\frac{a}{ea-db}\right)} \begin{pmatrix} 1 & \frac{b}{a} & \frac{c}{a} & \frac{1}{a} & 0 & 0 \\ 0 & 1 & \frac{fa-dc}{ea-db} & \frac{-d}{ea-db} & \frac{a}{ea-db} & 0 \\ 0 & \frac{-bp+aq}{a} & \frac{-cp+ar}{a} & \frac{-p}{a} & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line2} \times \left(\frac{-b}{a}\right)}$$

$$\begin{pmatrix} 1 & 0 & \frac{-fb+ec}{ea-db} & \frac{e}{ea-db} & \frac{-b}{ea-db} & 0 \\ 0 & 1 & \frac{fa-dc}{ea-db} & \frac{-d}{ea-db} & \frac{a}{ea-db} & 0 \\ 0 & \frac{-bp+aq}{a} & \frac{-cp+ar}{a} & \frac{-p}{a} & 0 & 1 \end{pmatrix}$$

$$\xrightarrow{\text{line3 } += \text{ line2} \times \left(\frac{bp-aq}{a}\right)}$$

$$\begin{pmatrix} 1 & 0 & \frac{-fb+ec}{ea-db} & \frac{e}{ea-db} & \frac{-b}{ea-db} & 0 \\ 0 & 1 & \frac{fa-dc}{ea-db} & \frac{-d}{ea-db} & \frac{a}{ea-db} & 0 \\ 0 & 0 & \frac{(fb-ec)p+(-fa+dc)q+(ea-db)r}{ea-db} & \frac{-ep+dq}{ea-db} & \frac{bp-aq}{ea-db} & 1 \end{pmatrix}$$

If $r = \dfrac{(-fb+ec)p+(fa-dc)q}{ea-db},$

$$\begin{pmatrix} 1 & 0 & \frac{-fb+ec}{ea-db} & \frac{e}{ea-db} & \frac{-b}{ea-db} & 0 \\ 0 & 1 & \frac{fa-dc}{ea-db} & \frac{-d}{ea-db} & \frac{a}{ea-db} & 0 \\ 0 & 0 & 0 & \frac{-ep+dq}{ea-db} & \frac{bp-aq}{ea-db} & 1 \end{pmatrix}$$

If $(fb-ec)p+(-fa+dc)q+(ea-db)r \neq 0,$

$$\xrightarrow{\text{line3 } \times= \left(\frac{ea-db}{(fb-ec)p+(-fa+dc)q+(ea-db)r}\right)}$$

$$\begin{pmatrix} 1 & 0 & \frac{-fb+ec}{ea-db} & \frac{e}{ea-db} & \frac{-b}{ea-db} & 0 \\ 0 & 1 & \frac{fa-dc}{ea-db} & \frac{-d}{ea-db} & \frac{a}{ea-db} & 0 \\ 0 & 0 & 1 & \frac{-ep+dq}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{bp-aq}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{ea-db}{(fb-ec)p+(-fa+dc)q+(ea-db)r} \end{pmatrix}$$

$$\xrightarrow{\text{line1 } += \text{ line3} \times \left(\frac{fb-ec}{ea-db}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{-fq+er}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{cq-br}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{fb-ec}{(fb-ec)p+(-fa+dc)q+(ea-db)r} \\ 0 & 1 & \frac{fa-dc}{ea-db} & \frac{-d}{ea-db} & \frac{a}{ea-db} & 0 \\ 0 & 0 & 1 & \frac{-ep+dq}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{bp-aq}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{ea-db}{(fb-ec)p+(-fa+dc)q+(ea-db)r} \end{pmatrix}$$

$$\xrightarrow{\text{line2 } += \text{ line3} \times \left(\frac{-fa+dc}{ea-db}\right)}$$

$$\begin{pmatrix} 1 & 0 & 0 & \frac{-fq+er}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{cq-br}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{fb-ec}{(fb-ec)p+(-fa+dc)q+(ea-db)r} \\ 0 & 1 & 0 & \frac{fp-dr}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{-cp+ar}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{-fa+dc}{(fb-ec)p+(-fa+dc)q+(ea-db)r} \\ 0 & 0 & 1 & \frac{-ep+dq}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{bp-aq}{(fb-ec)p+(-fa+dc)q+(ea-db)r} & \frac{ea-db}{(fb-ec)p+(-fa+dc)q+(ea-db)r} \end{pmatrix}$$

```
[10] os_md.mdivisor(mat([3,5,7],[5,3,3]),0|dviout=2)
```

$$\begin{pmatrix} 3 & 5 & 7 & 1 & 0 \\ 5 & 3 & 3 & 0 & 1 \\ 1 & 0 & 0 & & \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & & \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} L1 \\ L2 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 7 & 11 & 2 & -1 \\ 0 & -16 & -26 & -5 & 3 \\ 1 & 0 & 0 & & \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & & \end{pmatrix}$$

$$Cj \ -= \ C1 \times \circ \quad (j > 1)$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 2 & -1 \\ 0 & -16 & -26 & -5 & 3 \\ 1 & -7 & -11 & & \\ 0 & 1 & 0 & & \\ 0 & 0 & 1 & & \end{pmatrix}$$

$$\begin{pmatrix} C2 & C3 \end{pmatrix} \begin{pmatrix} -5 & -13 \\ 3 & 8 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & 2 & -1 \\ 0 & 2 & 0 & -5 & 3 \\ 1 & 2 & 3 & & \\ 0 & -5 & -13 & & \\ 0 & 3 & 8 & & \end{pmatrix}$$

As a result,

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 3 & 5 & 7 \\ 5 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -5 & -13 \\ 0 & 3 & 8 \end{pmatrix},$$

$$\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}^{-1} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -5 & -13 \\ 0 & 3 & 8 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 7 & 11 \\ 0 & -8 & -13 \\ 0 & 3 & 5 \end{pmatrix}.$$

```
[11] A=mat([2,-2,-2],[0,1,-1],[0,0,2])$
[12] os_md.mdivisor(os_md.mgen(3,0,[x],0)-A,x|dviout=2)$
```

$$\begin{pmatrix} x-2 & 2 & 2 & 1 & 0 & 0 \\ 0 & x-1 & 1 & 0 & 1 & 0 \\ 0 & 0 & x-2 & 0 & 0 & 1 \\ 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & & \\ 0 & 0 & 1 & & & \end{pmatrix}$$

$$C1 \leftrightarrow C2$$

$$\to \begin{pmatrix} 2 & x-2 & 2 & 1 & 0 & 0 \\ x-1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & x-2 & 0 & 0 & 1 \\ 0 & 1 & 0 & & & \\ 1 & 0 & 0 & & & \\ 0 & 0 & 1 & & & \end{pmatrix}$$

$$L1 \leftarrow (\frac{1}{2}) \times L1$$

$$\to \begin{pmatrix} 1 & \frac{1}{2}(x-2) & 1 & \frac{1}{2} & 0 & 0 \\ x-1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & x-2 & 0 & 0 & 1 \\ 0 & 1 & 0 & & & \\ 1 & 0 & 0 & & & \\ 0 & 0 & 1 & & & \end{pmatrix}$$

$$Li \ -= \ \circ \times L1 \quad (i > 1)$$

$$\to \begin{pmatrix} 1 & \frac{1}{2}(x-2) & 1 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2}(x-2)(x-1) & -(x-2) & -\frac{1}{2}(x-1) & 1 & 0 \\ 0 & 0 & x-2 & 0 & 0 & 1 \\ 0 & 1 & 0 & & & \\ 1 & 0 & 0 & & & \\ 0 & 0 & 1 & & & \end{pmatrix}$$

$Cj \ -= \ C1 \times \circ \quad (j > 1)$

$$\to \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2}(x-2)(x-1) & -(x-2) & -\frac{1}{2}(x-1) & 1 & 0 \\ 0 & 0 & x-2 & 0 & 0 & 1 \\ 0 & 1 & 0 & & & \\ 1 & -\frac{1}{2}(x-2) & -1 & & & \\ 0 & 0 & 1 & & & \end{pmatrix}$$

$L2 \leftrightarrow L3, \ C2 \leftrightarrow C3$

$$\to \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & x-2 & 0 & 0 & 0 & 1 \\ 0 & -(x-2) & -\frac{1}{2}(x-2)(x-1) & -\frac{1}{2}(x-1) & 1 & 0 \\ 0 & 0 & 1 & & & \\ 1 & -1 & -\frac{1}{2}(x-2) & & & \\ 0 & 1 & 0 & & & \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} L2 \\ L3 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & x-2 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{2}(x-2)(x-1) & -\frac{1}{2}(x-1) & 1 & 1 \\ 0 & 0 & 1 & & & \\ 1 & -1 & -\frac{1}{2}(x-2) & & & \\ 0 & 1 & 0 & & & \end{pmatrix}$$

$L3 \leftarrow (-2) \times L3$

$$\to \begin{pmatrix} 1 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & x-2 & 0 & 0 & 0 & 1 \\ 0 & 0 & (x-2)(x-1) & x-1 & -2 & -2 \\ 0 & 0 & 1 & & & \\ 1 & -1 & -\frac{1}{2}(x-2) & & & \\ 0 & 1 & 0 & & & \end{pmatrix}$$

As a result,

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & x-2 & 0 \\ 0 & 0 & (x-2)(x-1) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 \\ x-1 & -2 & -2 \end{pmatrix} \begin{pmatrix} x-2 & 2 & 2 \\ 0 & x-1 & 1 \\ 0 & 0 & x-2 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & -\frac{1}{2}(x-2) \\ 0 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 \\ x-1 & -2 & -2 \end{pmatrix}^{-1} = \begin{pmatrix} 2 & 0 & 0 \\ x-1 & -1 & -\frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix},$$

$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & -1 & -\frac{1}{2}(x-2) \\ 0 & 1 & 0 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{2}(x-2) & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

```
[13] os_md.mdivisor(os_md.mgen(2,0,[dx],0),[x,dx]|dviout=2);
```

$$\begin{pmatrix} \partial & 0 & 1 & 0 \\ 0 & \partial & 0 & 1 \\ 1 & 0 & & \\ 0 & 1 & & \end{pmatrix}$$

$C1 \ += \ C2 \times (x)$

$$\to \begin{pmatrix} \partial & 0 & 1 & 0 \\ x\partial + 1 & \partial & 0 & 1 \\ 1 & 0 & & \\ x & 1 & & \end{pmatrix}$$

$$\begin{pmatrix} -x & 1 \\ x\partial + 2 & -\partial \end{pmatrix} \begin{pmatrix} L1 \\ L2 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & \partial & -x & 1 \\ 0 & -\partial^2 & x\partial+2 & -\partial \\ 1 & 0 & & \\ x & 1 & & \end{pmatrix}$$

$Cj \mathrel{-}= C1 \times \circ \quad (j > 1)$

$$\to \begin{pmatrix} 1 & 0 & -x & 1 \\ 0 & -\partial^2 & x\partial+2 & -\partial \\ 1 & -\partial & & \\ x & -x\partial+1 & & \end{pmatrix}$$

$L2 \leftarrow (-1) \times L2$

$$\to \begin{pmatrix} 1 & 0 & -x & 1 \\ 0 & \partial^2 & -x\partial-2 & \partial \\ 1 & -\partial & & \\ x & -x\partial+1 & & \end{pmatrix}$$

As a result,

$$\begin{pmatrix} 1 & 0 \\ 0 & \partial^2 \end{pmatrix} = \begin{pmatrix} -x & 1 \\ -x\partial-2 & \partial \end{pmatrix} \begin{pmatrix} \partial & 0 \\ 0 & \partial \end{pmatrix} \begin{pmatrix} 1 & -\partial \\ x & -x\partial+1 \end{pmatrix},$$

$$\begin{pmatrix} -x & 1 \\ -x\partial-2 & \partial \end{pmatrix}^{-1} = \begin{pmatrix} \partial & -1 \\ x\partial+1 & -x \end{pmatrix},$$

$$\begin{pmatrix} 1 & -\partial \\ x & -x\partial+1 \end{pmatrix}^{-1} = \begin{pmatrix} -x\partial & \partial \\ -x & 1 \end{pmatrix}.$$

```
[14] os_md.mdivisor(mat([dx,0,0],[0,dx,0],[0,0,dx]),[x,dx]|dviout=3)
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \partial^3 \end{pmatrix} = P \begin{pmatrix} \partial & 0 & 0 \\ 0 & \partial & 0 \\ 0 & 0 & \partial \end{pmatrix} Q,$$

$$P = \begin{pmatrix} -x & 1 & 0 \\ -\frac{1}{2}x\partial-1 & \frac{1}{2}\partial & -\frac{1}{2}x^2\partial-2x \\ -x\partial^2-3\partial & \partial^2 & -x^2\partial^2-6x\partial-6 \end{pmatrix} = \begin{pmatrix} \partial & -x^2\partial^2-4x\partial-2 & \frac{1}{2}x^2\partial+2x \\ x\partial+1 & -x^3\partial^2-4x^2\partial-2x & \frac{1}{2}x^3\partial+2x^2 \\ 0 & \partial & \frac{-1}{2} \end{pmatrix}^{-1},$$

$$Q = \begin{pmatrix} 1 & -x^2\partial-2x & \frac{1}{2}x^2\partial^3+x\partial^2-\partial \\ x & -x^3\partial-x^2 & \frac{1}{2}x^3\partial^3+\frac{1}{2}x^2\partial^2-x\partial+1 \\ 0 & 1 & -\frac{1}{2}\partial^2 \end{pmatrix} = \begin{pmatrix} -x\partial & \partial & 0 \\ -\frac{1}{2}x\partial^2-\partial & \frac{1}{2}\partial^2 & -\frac{1}{2}x^2\partial^2-2x\partial \\ -x & 1 & -x^2 \end{pmatrix}^{-1}.$$