

$$\begin{aligned}\int \frac{bx^2 + ca}{cb} dx &= \frac{bx^3 + 3cax}{3cb} \\ &= \frac{1}{3c}x^3 + \frac{a}{b}x\end{aligned}$$

$$\int \exp(x)x^2 dx = (x^2 - 2x + 2) \exp(x)$$

$$\begin{aligned}\int a^x x dx &= \frac{a^x (\log(a)x - 1)}{(\log(a))^2} \\ &= \frac{\log(a)x - 1}{(\log(a))^2} a^x\end{aligned}$$

$$\begin{aligned}\int \sin(cx)x^2 dx &= (-\cos(cx)c^4x^2 + 2\sin(cx)c^3x + 2\cos(cx)c^2) / (c^5) \\ &= \frac{2x}{c^2} \cos(cx) \tan(cx) - \frac{c^2x^2 - 2}{c^3} \cos(cx)\end{aligned}$$

$$\int 4(\log|x+1|)^2 x dx = 2(x-1)(x+1)(\log|x+1|)^2 - 2(x-3)(x+1)\log|x+1| + x(x-6)$$

$$\int \frac{2dx}{(x^2 + a^2)^2} = \frac{1}{a^3} \arctan\left(\frac{x}{a}\right) + \frac{x}{a^2(x^2 + a^2)}$$

$$\begin{aligned}\int \frac{4dx}{(x-a)^2(x+a)^2} &= \int \left(\frac{1}{a^3(x+a)} + \frac{1}{a^2(x+a)^2} + \frac{-1}{a^3(x-a)} + \frac{1}{a^2(x-a)^2} \right) dx \\ &= \frac{\log|x+a|}{a^3} + \int \left(\frac{1}{a^2(x+a)^2} + \frac{-1}{a^3(x-a)} + \frac{1}{a^2(x-a)^2} \right) dx \\ &= \frac{\log|x+a|x + (\log|x+a| - 1)a}{a^3(x+a)} + \int \left(\frac{-1}{a^3(x-a)} + \frac{1}{a^2(x-a)^2} \right) dx \\ &= \frac{(\log|x+a| - \log|x-a|)x + (\log|x+a| - \log|x-a| - 1)a}{a^3(x+a)} + \int \frac{1}{a^2(x-a)^2} dx \\ &= ((\log|x+a| - \log|x-a|)x^2 - 2ax + (-\log|x+a| + \log|x-a|)a^2) / (a^3x^2 - a^5) \\ &= \frac{1}{a^3} \log \left| \frac{x+a}{x-a} \right| - \frac{2x}{a^2(x-a)(x+a)}\end{aligned}$$

$$\int \frac{dx}{x^2 - 2} = -\frac{1}{4}\sqrt{2} \log \left| \frac{x + \sqrt{2}}{x - \sqrt{2}} \right|$$

$$\begin{aligned}\int \frac{dx}{x^4 + 1} &= \int \left(\frac{x + \sqrt{2}}{2\sqrt{2}(x^2 + \sqrt{2}x + 1)} + \frac{-(x - \sqrt{2})}{2\sqrt{2}(x^2 - \sqrt{2}x + 1)} \right) dx \\ &= \frac{1}{8} \left(\log(x^2 + \sqrt{2}x + 1) + 2 \arctan(\sqrt{2}x + 1) \right) \sqrt{2} + \int \frac{-(x - \sqrt{2})}{2\sqrt{2}(x^2 - \sqrt{2}x + 1)} dx \\ &= \frac{1}{8} \left(\log(x^2 + \sqrt{2}x + 1) + 2 \arctan(\sqrt{2}x + 1) - \log(x^2 - \sqrt{2}x + 1) + 2 \arctan(\sqrt{2}x - 1) \right) \sqrt{2} \\ &= \frac{1}{4}\sqrt{2} \arctan(\sqrt{2}x + 1) + \frac{1}{4}\sqrt{2} \arctan(\sqrt{2}x - 1) + \frac{1}{8}\sqrt{2} \log \left(\frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1} \right)\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{x^6+1} &= \int \left(\frac{\sqrt{3}x+2}{6(x^2+\sqrt{3}x+1)} + \frac{-(\sqrt{3}x-2)}{6(x^2-\sqrt{3}x+1)} + \frac{1}{3(x^2+1)} \right) dx \\
&= \frac{1}{12} \left(\log|x^2+\sqrt{3}x+1| \sqrt{3} + 2 \arctan(2x+\sqrt{3}) \right) + \int \left(\frac{-(\sqrt{3}x-2)}{6(x^2-\sqrt{3}x+1)} + \frac{1}{3(x^2+1)} \right) dx \\
&= \frac{1}{12} \left(\left(\log|x^2+\sqrt{3}x+1| - \log|x^2-\sqrt{3}x+1| \right) \sqrt{3} + 2 \arctan(2x+\sqrt{3}) + 2 \arctan(2x-\sqrt{3}) \right) + \int \frac{1}{3(x^2+1)} dx \\
&= \frac{1}{12} \left(\left(\log|x^2+\sqrt{3}x+1| - \log|x^2-\sqrt{3}x+1| \right) \sqrt{3} + 2 \arctan(2x+\sqrt{3}) + 2 \arctan(2x-\sqrt{3}) + 4 \arctan(x) \right) \\
&= \frac{1}{6} \arctan(2x+\sqrt{3}) + \frac{1}{6} \arctan(2x-\sqrt{3}) + \frac{1}{3} \arctan(x) + \frac{1}{12} \sqrt{3} \log \left| \frac{x^2+\sqrt{3}x+1}{x^2-\sqrt{3}x+1} \right|
\end{aligned}$$

$$\begin{aligned}
\int 2 \exp(x) \sin(x) x dx &= \exp(x) ((\sin(x) - \cos(x)) x + \cos(x)) \\
&= x \tan(x) \cos(x) \exp(x) - (x-1) \cos(x) \exp(x)
\end{aligned}$$

$$\int \tan(x) dx = -\log|\cos(x)|$$

$$\begin{aligned}
\int 2 \tan(2x) \sin(2x) dx &\quad (t = \tan(x)) \\
&= \int \frac{-8t^2}{t^6+t^4-t^2-1} dt \\
&= \int \left(\frac{2}{t^2+1} + \frac{-4}{(t^2+1)^2} + \frac{1}{t+1} + \frac{-1}{t-1} \right) dt \\
&= 2 \arctan(t) + \int \left(\frac{-4}{(t^2+1)^2} + \frac{1}{t+1} + \frac{-1}{t-1} \right) dt \\
&= \frac{-2t}{t^2+1} + \int \left(\frac{1}{t+1} + \frac{-1}{t-1} \right) dt \\
&= \frac{\log|t+1|t^2-2t + \log|t+1|}{t^2+1} + \int \frac{-1}{t-1} dt \\
&= \frac{(\log|t+1| - \log|t-1|)t^2-2t + \log|t+1| - \log|t-1|}{t^2+1} \\
&= \frac{\log \left| \frac{t+1}{t-1} \right| t^2-2t + \log \left| \frac{t+1}{t-1} \right|}{t^2+1} \\
&= \log \left| \frac{t+1}{t-1} \right| + \frac{-2t}{t^2+1} \\
&= \log \left| \frac{\tan(x)+1}{\tan(x)-1} \right| + \frac{-2t}{t^2+1} \\
&= \log \left| \frac{\tan(x)+1}{\tan(x)-1} \right| + (-\sin(2x)) \\
&= -\sin(2x) + \log \left| \frac{\tan(x)+1}{\tan(x)-1} \right|
\end{aligned}$$

$$\begin{aligned}
\int (\tan(x))^2 dx & \quad (t = \tan(x)) \\
&= \int \frac{t^2}{t^2 + 1} dt \\
&= \int \left(1 + \frac{-1}{t^2 + 1}\right) dt \\
&= t + \int \frac{-1}{t^2 + 1} dt \\
&= t - \arctan(t) \\
&= -x + \frac{\sin(x)}{\cos(x)} \\
&= \frac{-\cos(x)x + \sin(x)}{\cos(x)} \\
&= \tan(x) - x
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{4 \sin(x) + 3 \cos(x)} & \quad (t = \tan(\frac{1}{2}x)) \\
&= \int \frac{-2}{3t^2 - 8t - 3} dt \\
&= \int \left(\frac{3}{5(3t + 1)} + \frac{-1}{5(t - 3)}\right) dt \\
&= \frac{1}{5} \log |3t + 1| + \int \frac{-1}{5(t - 3)} dt \\
&= \frac{1}{5} (\log |3t + 1| - \log |t - 3|) \\
&= \frac{1}{5} \log \left| \frac{3t + 1}{t - 3} \right| \\
&= \frac{1}{5} \log \left| \frac{3 \tan(\frac{1}{2}x) + 1}{\tan(\frac{1}{2}x) - 3} \right|
\end{aligned}$$

$$\begin{aligned}
\int \arctan(x) dx &= \arctan(x)x + \int \frac{-x}{x^2 + 1} dx \\
&= \arctan(x)x + \left(-\frac{1}{2} \log(x^2 + 1)\right) \\
&= x \arctan(x) - \frac{1}{2} \log(x^2 + 1)
\end{aligned}$$

$$\begin{aligned}
\int \left(6 \arctan\left(\frac{1}{x}\right) x^2 + 6 \arctan\left(\frac{1}{x}\right)\right) dx &= 2 \arctan\left(\frac{1}{x}\right) x(x^2 + 3) + \int \frac{2x(x^2 + 3)}{x^2 + 1} dx \\
&= 2 \arctan\left(\frac{1}{x}\right) x(x^2 + 3) + \int \frac{2x^3 + 6x}{x^2 + 1} dx \\
&= 2 \arctan\left(\frac{1}{x}\right) x(x^2 + 3) + \int \left(\frac{2x}{1} + \frac{4x}{x^2 + 1}\right) dx \\
&= 2 \arctan\left(\frac{1}{x}\right) x(x^2 + 3) + x^2 + \int \frac{4x}{x^2 + 1} dx \\
&= 2 \arctan\left(\frac{1}{x}\right) x(x^2 + 3) + x^2 + 2 \log(x^2 + 1) \\
&= 2 \log(x^2 + 1) + 2x(x^2 + 3) \arctan\left(\frac{1}{x}\right) + x^2
\end{aligned}$$

$$\begin{aligned}
\int (-a^2x^2 + 2bx)^{-\frac{1}{2}} dx & \quad \left(\frac{a^2x - b}{a} = \frac{\sin(t)b}{a}\right) \\
&= \int \frac{1}{a} dt \\
&= \frac{t}{a} \\
&= \frac{1}{a} \arcsin\left(\frac{a^2x - b}{b}\right)
\end{aligned}$$

$$\begin{aligned}
\int \sqrt{-x^2 + a^2} dx & \quad (x = \sin(t)a) \\
&= \int (\cos(t))^2 a^2 dt \\
&= \int \frac{1}{2} (\cos(2t) + 1) a^2 dt \\
&= \frac{1}{2} a^2 (t + \cos(t) \sin(t)) \\
&= \frac{1}{2} x \sqrt{-x^2 + a^2} + \frac{1}{2} a^2 \arcsin\left(\frac{x}{a}\right)
\end{aligned}$$

$$\begin{aligned}
\int (x^2 - 1)^{-\frac{1}{2}} dx & \quad \left(x = \frac{1}{\cos(t)}\right) \\
&= \int \frac{1}{\cos(t)} dt \\
& \quad (s = \tan\left(\frac{1}{2}t\right)) \\
&= \int \frac{-2}{s^2 - 1} ds \\
&= \int \left(\frac{1}{s+1} + \frac{-1}{s-1}\right) ds \\
&= \log|s+1| + \int \frac{-1}{s-1} ds \\
&= \log|s+1| - \log|s-1| \\
&= \log\left|\frac{s+1}{s-1}\right| \\
&= \log\left|\frac{\tan\left(\frac{1}{2}t\right) + 1}{\tan\left(\frac{1}{2}t\right) - 1}\right| \\
&= \log\left|\frac{\sin(t) + \cos(t) + 1}{\sin(t) - \cos(t) - 1}\right| \\
&= \log\left|x + \sqrt{x^2 - 1}\right|
\end{aligned}$$

$$\begin{aligned}
& \int \sqrt{x^2 - 1} \, dx \quad \left(x = \frac{1}{\cos(t)} \right) \\
&= \int \frac{(\sin(t))^2}{(\cos(t))^3} dt \\
&\quad (s = \tan(\frac{1}{2}t)) \\
&= \int \frac{-8s^2}{(s-1)^3(s+1)^3} ds \\
&= \int \left(\frac{-1}{2(s+1)} + \frac{-1}{2(s+1)^2} + \frac{1}{(s+1)^3} + \frac{1}{2(s-1)} + \frac{-1}{2(s-1)^2} + \frac{-1}{(s-1)^3} \right) ds \\
&= -\frac{1}{2} \log|s+1| + \int \left(\frac{-1}{2(s+1)^2} + \frac{1}{(s+1)^3} + \frac{1}{2(s-1)} + \frac{-1}{2(s-1)^2} + \frac{-1}{(s-1)^3} \right) ds \\
&= \frac{-(\log|s+1|s + \log|s+1| - 1)}{2(s+1)} + \int \left(\frac{1}{(s+1)^3} + \frac{1}{2(s-1)} + \frac{-1}{2(s-1)^2} + \frac{-1}{(s-1)^3} \right) ds \\
&= \frac{-(\log|s+1|s^2 + (2\log|s+1| - 1)s + \log|s+1|)}{2(s+1)^2} + \int \left(\frac{1}{2(s-1)} + \frac{-1}{2(s-1)^2} + \frac{-1}{(s-1)^3} \right) ds \\
&= \frac{-(\log|s+1| - \log|s-1|)s^2 + (2\log|s+1| - 2\log|s-1| - 1)s + \log|s+1| - \log|s-1|}{2(s+1)^2} + \int \left(\frac{-1}{2(s-1)^2} + \frac{-1}{(s-1)^3} \right) ds \\
&= \frac{-(\log|s+1| - \log|s-1|)s^3 + (\log|s+1| - \log|s-1| - 2)s^2 + (-\log|s+1| + \log|s-1| - 1)s - \log|s+1| + \log|s-1|}{2(s-1)(s+1)^2} \\
&= \frac{-(\log|s+1| - \log|s-1|)s^4 - 2s^3 + (-2\log|s+1| + 2\log|s-1|)s^2 - 2s + \log|s+1| - \log|s-1|}{2(s-1)^2(s+1)^2} \\
&= \frac{\log\left|\frac{s-1}{s+1}\right|s^4 + 2s^3 - 2\log\left|\frac{s-1}{s+1}\right|s^2 + 2s + \log\left|\frac{s-1}{s+1}\right|}{2(s-1)^2(s+1)^2} \\
&= \frac{1}{2} \log\left|\frac{s-1}{s+1}\right| + \frac{s^3 + s}{s^4 - 2s^2 + 1} \\
&= -\frac{1}{2} \log\left|\frac{\tan(\frac{1}{2}t) + 1}{\tan(\frac{1}{2}t) - 1}\right| + \frac{s^3 + s}{s^4 - 2s^2 + 1} \\
&= -\frac{1}{2} \log\left|\frac{\tan(\frac{1}{2}t) + 1}{\tan(\frac{1}{2}t) - 1}\right| + \frac{\sin(t)}{\cos(2t) + 1} \\
&= \frac{2\sin(t) - \log\left|\frac{\tan(\frac{1}{2}t) + 1}{\tan(\frac{1}{2}t) - 1}\right| \cos(2t) - \log\left|\frac{\tan(\frac{1}{2}t) + 1}{\tan(\frac{1}{2}t) - 1}\right|}{2(\cos(2t) + 1)} \\
&= \frac{\sin(t) - \log\left|\frac{\sin(\frac{1}{2}t) + \cos(\frac{1}{2}t)}{\sin(\frac{1}{2}t) - \cos(\frac{1}{2}t)}\right| (\cos(t))^2}{2(\cos(t))^2} \\
&= \frac{1}{2} x \sqrt{x^2 - 1} - \frac{1}{2} \log|x + \sqrt{x^2 - 1}|
\end{aligned}$$

$$\begin{aligned}
\int (x^2 + a^2)^{-\frac{1}{2}} dx & \quad \left(x = \frac{\sin(t)a}{\cos(t)} \right) \\
&= \int \frac{1}{\cos(t)} dt \\
& \quad (s = \tan(\frac{1}{2}t)) \\
&= \int \frac{-2}{s^2 - 1} ds \\
&= \int \left(\frac{1}{s+1} + \frac{-1}{s-1} \right) ds \\
&= \log |s+1| + \int \frac{-1}{s-1} ds \\
&= \log |s+1| - \log |s-1| \\
&= \log \left| \frac{s+1}{s-1} \right| \\
&= \log \left| \frac{\tan(\frac{1}{2}t) + 1}{\tan(\frac{1}{2}t) - 1} \right| \\
&= \log \left| \frac{\sin(t) + \cos(t) + 1}{\sin(t) - \cos(t) - 1} \right| \\
&= \log \left| x + \sqrt{x^2 + a^2} \right|
\end{aligned}$$

$$\begin{aligned}
& \int 2\sqrt{x^2 + a^2} dx \quad \left(x = \frac{\sin(t)a}{\cos(t)} \right) \\
&= \int \frac{2a^2}{(\cos(t))^3} dt \\
&\quad (s = \tan(\frac{1}{2}t)) \\
&= \int \frac{-4a^2 (s^2 + 1)^2}{(s-1)^3(s+1)^3} ds \\
&= \int \left(\frac{a^2}{s+1} + \frac{-a^2}{(s+1)^2} + \frac{2a^2}{(s+1)^3} + \frac{-a^2}{s-1} + \frac{-a^2}{(s-1)^2} + \frac{-2a^2}{(s-1)^3} \right) ds \\
&= \log|s+1|a^2 + \int \left(\frac{-a^2}{(s+1)^2} + \frac{2a^2}{(s+1)^3} + \frac{-a^2}{s-1} + \frac{-a^2}{(s-1)^2} + \frac{-2a^2}{(s-1)^3} \right) ds \\
&= \frac{a^2(\log|s+1|s + \log|s+1| + 1)}{s+1} + \int \left(\frac{2a^2}{(s+1)^3} + \frac{-a^2}{s-1} + \frac{-a^2}{(s-1)^2} + \frac{-2a^2}{(s-1)^3} \right) ds \\
&= \frac{a^2(\log|s+1|s^2 + (2\log|s+1| + 1)s + \log|s+1|)}{(s+1)^2} + \int \left(\frac{-a^2}{s-1} + \frac{-a^2}{(s-1)^2} + \frac{-2a^2}{(s-1)^3} \right) ds \\
&= \frac{a^2((\log|s+1| - \log|s-1|)s^2 + (2\log|s+1| - 2\log|s-1| + 1)s + \log|s+1| - \log|s-1|)}{(s+1)^2} + \int \left(\frac{-a^2}{(s-1)^2} + \frac{-2a^2}{(s-1)^3} \right) ds \\
&= \frac{a^2((\log|s+1| - \log|s-1|)s^3 + (\log|s+1| - \log|s-1| + 2)s^2 + (-\log|s+1| + \log|s-1| + 1)s - \log|s+1| + \log|s-1|)}{(s-1)(s+1)^2} \\
&= \frac{a^2((\log|s+1| - \log|s-1|)s^4 + 2s^3 + (-2\log|s+1| + 2\log|s-1|)s^2 + 2s + \log|s+1| - \log|s-1|)}{(s-1)^2(s+1)^2} \\
&= \frac{a^2 \left(\log \left| \frac{s+1}{s-1} \right| s^4 + 2s^3 - 2 \log \left| \frac{s+1}{s-1} \right| s^2 + 2s + \log \left| \frac{s+1}{s-1} \right| \right)}{(s-1)^2(s+1)^2} \\
&= \log \left| \frac{s+1}{s-1} \right| a^2 + \frac{2a^2s^3 + 2a^2s}{s^4 - 2s^2 + 1} \\
&= \log \left| \frac{\tan(\frac{1}{2}t) + 1}{\tan(\frac{1}{2}t) - 1} \right| a^2 + \frac{2a^2s^3 + 2a^2s}{s^4 - 2s^2 + 1} \\
&= \log \left| \frac{\tan(\frac{1}{2}t) + 1}{\tan(\frac{1}{2}t) - 1} \right| a^2 + \frac{2\sin(t)a^2}{\cos(2t) + 1} \\
&= \left(\left(2\sin(t) + \log \left| \frac{\tan(\frac{1}{2}t) + 1}{\tan(\frac{1}{2}t) - 1} \right| \cos(2t) + \log \left| \frac{\tan(\frac{1}{2}t) + 1}{\tan(\frac{1}{2}t) - 1} \right| \right) a^2 \right) / (\cos(2t) + 1) \\
&= \left(\left(\sin(t) + \log \left| \frac{\sin(\frac{1}{2}t) + \cos(\frac{1}{2}t)}{\sin(\frac{1}{2}t) - \cos(\frac{1}{2}t)} \right| (\cos(t))^2 \right) a^2 \right) / ((\cos(t))^2) \\
&= x\sqrt{x^2 + a^2} + a^2 \log \left| x + \sqrt{x^2 + a^2} \right|
\end{aligned}$$

$$\begin{aligned}
& \int \frac{\sqrt{x}}{x+1} dx \quad (t = \sqrt{x}) \\
&= \int \frac{2t^2}{t^2+1} dt \\
&= \int \left(2 + \frac{-2}{t^2+1} \right) dt \\
&= 2t + \int \frac{-2}{t^2+1} dt \\
&= 2\sqrt{x} - 2 \arctan(\sqrt{x})
\end{aligned}$$

$$\begin{aligned}
\int \frac{\sqrt[3]{x}}{x+1} dx & \quad (t = \sqrt[3]{x}) \\
&= \int \frac{3t^3}{t^3+1} dt \\
&= \int \left(3 + \frac{t-2}{t^2-t+1} + \frac{-1}{t+1} \right) dt \\
&= 3t + \int \left(\frac{t-2}{t^2-t+1} + \frac{-1}{t+1} \right) dt \\
&= 3t - \arctan\left(\frac{2}{3}\sqrt{3}t - \frac{1}{3}\sqrt{3}\right) \sqrt{3} + \frac{1}{2} \log|t^2-t+1| + \int \frac{-1}{t+1} dt \\
&= 3\sqrt[3]{x} - \sqrt{3} \arctan\left(\left(\frac{2}{3}\sqrt[3]{x} - \frac{1}{3}\right)\sqrt{3}\right) - \log|\sqrt[3]{x}+1| + \frac{1}{2} \log|(\sqrt[3]{x})^2 - \sqrt[3]{x}+1|
\end{aligned}$$

$$\begin{aligned}
\int \sqrt{\frac{-x+2}{x}} dx & \quad \left(t = \sqrt{\frac{-x+2}{x}}\right) \\
&= \int \frac{-4t^2}{(t^2+1)^2} dt \\
&= \int \left(\frac{-4}{t^2+1} + \frac{4}{(t^2+1)^2} \right) dt \\
&= -4 \arctan(t) + \int \frac{4}{(t^2+1)^2} dt \\
&= x \sqrt{\frac{-x+2}{x}} - 2 \arctan\left(\sqrt{\frac{-x+2}{x}}\right)
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{\exp(x) + \exp(-x)} & \quad (t = \exp(x)) \\
&= \int \frac{1}{t^2+1} dt \\
&= \arctan(\exp(x))
\end{aligned}$$

$$\begin{aligned}
\int \frac{\sin(x)x + \cos(x)}{\cos(x)x} dx &= \int \left(\frac{\sin(x)}{\cos(x)} + \frac{1}{x} \right) dx \\
&= -\log|\cos(x)| + \int \frac{1}{x} dx \\
&= \log|x| - \log|\cos(x)|
\end{aligned}$$

$$\begin{aligned}
\int \frac{dx}{x^4+4} &= \int \left(\frac{x+2}{8(x^2+2x+2)} + \frac{-(x-2)}{8(x^2-2x+2)} \right) dx \\
&= \frac{1}{16} (\log(x^2+2x+2) + 2 \arctan(x+1)) + \int \frac{-(x-2)}{8(x^2-2x+2)} dx \\
&= \frac{1}{16} (\log(x^2+2x+2) + 2 \arctan(x+1) - \log(x^2-2x+2) + 2 \arctan(x-1)) \\
&= \frac{1}{8} \arctan(x+1) + \frac{1}{8} \arctan(x-1) + \frac{1}{16} \log\left(\frac{x^2+2x+2}{x^2-2x+2}\right)
\end{aligned}$$

$$\begin{aligned}
\int \left(\frac{-\cos(x) + 1}{-\cos(x) + \frac{1}{2}} \right)^{\frac{1}{2}} dx & \quad (t = \tan(\frac{1}{2}x)) \\
&= \int \frac{4\sqrt{\frac{1}{3t^2-1}}t}{t^2+1} dt \\
& \quad \left(\sqrt{3}t = \frac{1}{\cos(s)} \right) \\
&= \int \frac{-4(\sin(s))^2}{3(\cos(s))^4 - 2(\cos(s))^2 - 1} ds \\
& \quad (u = \tan(s)) \\
&= \int \frac{4}{u^2+4} du \\
&= 2 \arctan\left(\frac{1}{2}u\right) \\
&= 2 \arctan\left(\frac{\frac{1}{2}\sin(s)}{\cos(s)}\right) \\
&= 2 \arctan\left(\frac{1}{2}\sqrt{3t^2-1}\right) \\
&= 2 \arctan\left(\frac{1}{2}\sqrt{3(\tan(\frac{1}{2}x))^2-1}\right)
\end{aligned}$$

$$\begin{aligned}
\int \log(x^2+1) dx &= \log(x^2+1)x + \int \frac{-2x^2}{x^2+1} dx \\
&= \log(x^2+1)x + \int \left((-2) + \frac{2}{x^2+1} \right) dx \\
&= \log(x^2+1)x + (-2x) + \int \frac{2}{x^2+1} dx \\
&= \log(x^2+1)x + (-2x + 2 \arctan(x)) \\
&= 2 \arctan(x) + x \log(x^2+1) - 2x
\end{aligned}$$

$$\begin{aligned}
\int \arctan(\sqrt{x}) dx &= \arctan(\sqrt{x})x + \int \frac{-x^{-\frac{1}{2}}x}{2((\sqrt{x})^2+1)} dx \\
&= \arctan(\sqrt{x})x + \int \frac{-\sqrt{x}}{2(x+1)} dx \\
& \quad (t = \sqrt{x}) \\
&= \arctan(\sqrt{x})x + \int \frac{-t^2}{t^2+1} dt \\
&= \arctan(\sqrt{x})x + \int \left((-1) + \frac{1}{t^2+1} \right) dt \\
&= \arctan(\sqrt{x})x + (-t) + \int \frac{1}{t^2+1} dt \\
&= \arctan(\sqrt{x})x + (-\sqrt{x} + \arctan(\sqrt{x})) \\
&= -\sqrt{x} + (x+1) \arctan(\sqrt{x})
\end{aligned}$$

$$\begin{aligned}
\int (\arcsin(x))^2 dx &= (\arcsin(x))^2 x + \int \left(-2(-x^2+1)^{-\frac{1}{2}} \arcsin(x)x \right) dx \\
&= (\arcsin(x))^2 x + 2\sqrt{-x^2+1} \arcsin(x) + \int \left(-2\sqrt{-x^2+1} (-x^2+1)^{-\frac{1}{2}} \right) dx \\
&= \arcsin(x) \left(\arcsin(x)x + 2\sqrt{-x^2+1} \right) + \int (-2) dx \\
&= \arcsin(x) \left(\arcsin(x)x + 2\sqrt{-x^2+1} \right) + (-2x) \\
&= x \arcsin(x)^2 + 2 \arcsin(x) \sqrt{-x^2+1} - 2x
\end{aligned}$$

$$\begin{aligned}
\int \frac{\log |x| (x + \log |x| + 1)}{x^2} dx &= \frac{-(\log |x|)^2}{x} + \int \frac{\log |x| (x + 3)}{x^2} dx \\
&= \frac{-(\log |x|)^2}{x} + \frac{\log |x| (\log |x|x - 6)}{2x} + \int \frac{3}{x^2} dx \\
&= \frac{\log |x| (\log |x|x - 2 \log |x| - 6)}{2x} + \int \frac{3}{x^2} dx \\
&= \frac{\log |x| (\log |x|x - 2 \log |x| - 6)}{2x} + \frac{-3}{x} \\
&= \frac{x-2}{2x} (\log |x|)^2 - \frac{3}{x} \log |x| - \frac{3}{x}
\end{aligned}$$

$$\int \frac{dx}{\log |x|x} = \log |\log |x||$$