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ELSEVIER Stochastic Processes and their Applications 115 (2005) 1167–1186
www.elsevier.com/locate/spa

Asymptotic expansion for Barndorff-Nielsen and Shephard's stochastic volatility model ☆

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Received 20 September 2003; received in revised form 22 November 2004; accepted 16 February 2005
Available online 17 March 2005

Abstract

With the help of a general methodology of asymptotic expansions for mixing processes, we obtain the Edgeworth expansion for log-returns of a stock price process in Barndorff-Nielsen and Shephard's stochastic volatility model, in which the latent volatility process is described by a stationary non-Gaussian Ornstein–Uhlenbeck process (OU process) with invariant selfdecomposable distribution on $\mathbb{R}_+$. The present result enables us to simultaneously explain non-Gaussianity for short time-lags as well as approximate Gaussianity for long time-lags. The Malliavin calculus formulated by Bichteler, Gravereaux and Jacod for processes with jumps and the exponential mixing property of the OU process play substantial roles in order to ensure a conditional type Cramér condition under a certain truncation. Owing to several inherent properties of OU processes, the regularity conditions for the expansions can be verified without any difficulty, and the coefficients of the expansions up to any order can be explicitly computed.

Keywords: Edgeworth expansion; Lévy process; Mixing; Non-Gaussian Ornstein–Uhlenbeck process; Stochastic volatility model

☆This work was in part supported by Cooperative Research Program 16-2002 of the Institute of Statistical Mathematics, and 21st Century COE Program "Development of Dynamic Mathematics with High Functionality" of Kyushu University.

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0304-4149/$ - see front matter
doi:10.1016/j.spa.2005.02.007

1. Introduction

In this paper we are concerned with the model $(X,Y)=\{(X_t,Y_t)\}_{t\in\mathbb{R}_+}$ given by

$$\begin{aligned}&\mathrm{d}X_t=-\lambda X_t\,\mathrm{d}t+\mathrm{d}Z_t,\\&\mathrm{d}Y_t=(\gamma+\beta X_t)\,\mathrm{d}t+\sqrt{X_t}\,\mathrm{d}w_t+\rho\,\mathrm{d}Z_t,\quad Y_0=0,\end{aligned}\tag{1}$$

where $Z=(Z_t)_{t\in\mathbb{R}_+}$ and $w=(w_t)_{t\in\mathbb{R}_+}$, respectively, denote a subordinator (increasing Lévy process) and a Wiener process independent of Z, and $(\lambda,\gamma,\beta,\rho)\in(0,\infty)\times\mathbb{R}^3$ are constants. The process X is called an Ornstein–Uhlenbeck process (OU process), where the initial variable X_0 is supposed to be independent of (Z,w). The positivity of λ together with a mild condition on the Lévy measure of Z ensures existence of an invariant distribution of X, which is necessarily selfdecomposable. The purpose of this paper is to obtain the Edgeworth expansion of an expectation $E[f(T^{-1/2}H_T)]$ as $T\to\infty$, where

$$\begin{aligned}H_T&:=Y_T-E[Y_T]\\&=\beta\int_0^T(X_s-E[X_0])\,\mathrm{d}s+\int_0^T\sqrt{X_s}\,\mathrm{d}w_s+\rho(Z_T-E[Z_T])\end{aligned}\tag{2}$$

and $f:\mathbb{R}\to\mathbb{R}$ is a measurable function at most polynomial growth; see Section 2.1 for precise formulation. We suppose that X is strictly stationary with a stationary distribution admitting moments of any order, and also that the Lévy measure of Z satisfies mild regularity conditions.

Eq. (1) is Barndorff-Nielsen and Shephard's continuous-time stochastic volatility model, in which X stays in $\mathbb{R}_+$ and describes a time-varying volatility: for clarity, we here do not adopt the unusual timing $\mathrm{d}Z_{\lambda t}$ for $\mathrm{d}Z_t$ of Barndorff-Nielsen and Shephard (in that case, a given marginal distribution of X is unchanged whatever $\lambda>0$ is); of course, this is not essential for validity of the expansion. This model not only captures several stylized features in finance and turbulence, but also offers a great deal of analytic tractability. See Barndorff-Nielsen and Shephard [5] and Barndorff-Nielsen [1] for details, and also Barndorff-Nielsen et al. [3] for a summary of recent developments in this direction. If X is ergodic, then the martingale central limit theorem yields that

$$\begin{aligned}&(E[X_0]T)^{-1/2}\left(Y_T-\gamma T-\beta\int_0^T X_s\,\mathrm{d}s-\rho(Z_T-E[Z_T])\right)\\&\quad=(E[X_0]T)^{-1/2}\int_0^T\sqrt{X_s}\,\mathrm{d}w_s\end{aligned}\tag{3}$$

weakly tends to the standard normal variable as $T\to\infty$. This is called "aggregational Gaussianity", which is recognized as one of important stylized features in turbulence as well as finance: here the ergodicity of X and exponential β-mixing property are indeed ensured by our Assumption 1 in Section 2.1 (cf. Masuda [15,16] for more general results). In this paper we consider the term "aggregational Gaussianity" as the central limit effect of the log-return $T^{-1/2}H_T$;

thus our setup in principle includes (3) with $\gamma = \beta = \rho = 0$. For real market data, it is quite well known that a distribution of log-returns exhibits non-Gaussianity for short time-lags and approximate Gaussianity for long time-lags. For this reason, it is interesting to investigate the higher order asymptotics of $\mathscr{L}(T^{-1/2}H_T)$ as well as its central limit effect for $T \to \infty$, so that we obtain a result which simultaneously explain non-Gaussianity for small T and approximated Gaussianity for large T.

To be convenient for readers, we now refer to some previous results concerning an OU process X whose solution is explicitly given by

$$X_t = \mathrm{e}^{-\lambda t} X_0 + \int_0^t \mathrm{e}^{-\lambda(t-s)}\, \mathrm{d}Z_s. \tag{4}$$

The corresponding references to the items (1)–(3) below can be found in Masuda [15, Section 2].

(1) Any selfdecomposable distribution, which is known to be unimodal and absolutely continuous with respect to the Lebesgue measure, can be realized as a stationary distribution of an OU process; more precisely, there is one-to-one correspondence between a possible stationary distribution of an OU process and a selfdecomposable distribution.

(2) Two theoretical construction of a stationary OU process with concrete marginal distribution are possible. First, suppose that a selfdecomposable distribution F is given. If $\varphi(u; F)$ is differentiable at $u \neq 0$ and moreover if the function $u \mapsto u\partial_u \kappa(u; F)$ is continuous at $u = 0$, then there exists a stationary OU process X with the marginal distribution F and Z determined by $\kappa(u; Z_1) = \lambda u \partial_u \kappa(u; F)$. Secondly, we can determine the stationary distribution F of X via a given generating triplet of Z_1; in this case, the Lévy measure $\Pi_Z(\mathrm{d}z)$ of Z must meet

$$\int_{|z|>1} \log |z|\, \Pi_Z(\mathrm{d}z) < \infty.$$

(3) If X is strictly stationary and the Lévy measure of F admits a differentiable density $g_F(x)$ for $x \neq 0$, then the Lévy measure of Z admits a density $g_Z(x)$ given by

$$g_Z(x) = -\lambda^{-1}\{g_F(x) + x\partial g_F(x)\}. \tag{5}$$

Relation (5) is convenient to determine Z, given F.

(4) If F (resp. Z_1) admits the kth cumulant, then Z_1 (resp. F) admits the kth cumulant as well and they are related by

$$k\lambda\kappa_F^{(k)} = \kappa_{Z_1}^{(k)}. \tag{6}$$

See Barndorff-Nielsen and Shephard [5, Section 2.1]; if we use Barndorff-Nielsen and Shephard's custom $\mathrm{d}Z_{\lambda t}$ instead of $\mathrm{d}Z_t$, then (6) becomes $k\kappa_F^{(k)} = \kappa_{Z_1}^{(k)}$.

An important and remarkable feature is that we can explicitly write down the coefficients of the asymptotic expansions up to any order, utilizing the relation

$$\int_0^t X_s\, \mathrm{d}s = \eta(\lambda, t) X_0 + \int_0^t \eta(\lambda, t-s)\, \mathrm{d}Z_s, \tag{7}$$

where $\eta(\lambda, u) = \lambda^{-1}(1 - \mathrm{e}^{-\lambda u})$: formula (7) directly follows from the explicit expression (4), or, the affine structure of the process, see Duffie et al. [11]. The formula (6) enables us to write down the coefficients of the asymptotic expansion in terms of only $\kappa_F^{(k)}$ or only $\kappa_{Z_1}^{(k)}$, $k \in \mathbb{N}$. One can consult Barndorff-Nielsen and Shephard [7] for a detailed analysis of integrated OU processes. Norberg [19] suggested the use of positive OU processes as a stochastic interest rate, and there moments of present values in actuarial context as well as price of zero-coupon bonds was studied, building on several explicit Laplace transforms concerning OU processes. See also Dassios and Jang [10, Section 2], where some conditional Laplace transforms of integrated positive processes of shot noise type were given.

Now let us observe that direct validation of the Edgeworth expansion, namely direct estimate of the characteristic function of $T^{-1/2}H_T$, is intractable. Lemma 3 below, which is more or less well known, and conditional argument (note that here X and w are independent) enable us to write down the characteristic function of $T^{-1/2}H_T$ as

$$\begin{aligned}\varphi(u; T^{-1/2}H_T) := {} & \exp\{-\mathrm{i}uT^{1/2}(\beta + \lambda\rho)E[X_0]\} \\ & \times E\left[\exp\left\{\left(\frac{\mathrm{i}u\beta}{T^{1/2}} - \frac{u^2}{2T}\right)\eta(\lambda, T)X_0\right\}\right] \\ & \times \exp\left\{\int_0^T \log E[\exp\{K(u,s)Z_1\}]\,\mathrm{d}s\right\}, \end{aligned} \tag{8}$$

where the complex-valued function K is given by

$$K(u,s) = \frac{\mathrm{i}u}{T^{1/2}}\{\rho + \beta\eta(\lambda, s)\} - \frac{u^2}{2T}\,\eta(\lambda, s)$$

whose real part is negative, so that $E[\exp\{K(u,s)Z_1\}]$ indeed exists since Z is a subordinator; see e.g. Sato [25, Theorem 30.1]. The most direct route to obtain the Edgeworth expansion is estimating $\varphi(u; T^{-1/2}H_T)$ for large $|u|$; this is called the "global approach" recently developed in Yoshida [27,28] covering processes with jumps. Unfortunately, the expression of $|\varphi(u; T^{-1/2}H_T)|$ involves the following rather intractable term coming from the Lévy-integral part in (8)

$$\left|\exp\left\{\int_0^T \int_{\mathbb{R}_+} \left(\exp\left[z\left\{\frac{\mathrm{i}u}{T^{1/2}}(\rho + \beta\eta(\lambda, s)) - \frac{u^2}{2T}\,\eta(\lambda, s)\right\}\right] - 1\right)\Pi_Z(\mathrm{d}z)\,\mathrm{d}s\right\}\right|,$$

where Π_Z denotes the Lévy measure of Z. Hence we shall take another route.

In this paper we are going to look at the "local approach", which is initiated by Götze and Hipp [12] recently extended to continuous-time framework by Yoshida [29]. [See also the previous works [13,23,24,29] for some statistical applications in this direction.] According to the Markov property of X as well as its exponential mixing property, this approach will turn out to be tailor-made for our aim. The main task is then to establish the following estimate for some $t^0, B > 0$, which results from the integration-by-parts formula:

$$E\left[\sup_{|u|\geqslant B}|E[\psi e^{iuH_{t^0}}|X_0, X_{t^0}]|\right]<1, \tag{9}$$

where ψ fulfilling $E[\psi]>0$ is a truncation functional, which enables us to extract a "nice event". Though (9), called the "conditional type Cramér condition", is generally not easy to verify, the concrete structure of the model (1) considerably simplifies the task. Also, the truncation technique is often inevitable, and this is indeed the case for our goal. In the proof we shall construct ψ in a tangible way in order to avoid the irregular square-root diffusion coefficient of Y, and consequently validate the expansion. See Section 4 for details.

The result is given in Section 2, and then Section 3 presents the explicit formulae for the asymptotic expansion. Section 4 devotes to proving the validity of the expansion.

2. Edgeworth expansion for log-returns

Let $(\Omega, \mathscr{F}, \mathbf{F} = (\mathscr{F}_t)_{t\in\mathbb{R}_+}, P)$ be a stochastic basis endowed with an $\mathbf{F}$-adapted non-trivial subordinator Z and an $\mathbf{F}$-adapted Wiener process w as well as an $\mathscr{F}_0$-measurable random variable X_0 independent of (w, Z). Throughout this article, $\varphi(u;\xi)$ stands for the characteristic function of ξ indicating a random variable or a distribution, and we write $\kappa(u;\xi) = \log\varphi(u;\xi)$ for the corresponding cumulant transform. The (partial) differentiation with respect to some variable v will be denoted by ∂_v, or simply by ∂ when there is no confusion.

Since Z is a subordinator, it can be completely characterized via a drift b_Z and a Lévy measure Π_Z, i.e.

$$\varphi(u;Z_t) = \exp\left\{t\left(ib_Z u + \int_{\mathbb{R}_+}(e^{iuz}-1)\,\Pi_Z(dz)\right)\right\},$$

where $b_Z\geqslant 0$, $\operatorname{supp}\Pi_Z\subset\mathbb{R}_+$ and $\int_{0<z\leqslant 1} z\,\Pi_Z(dz)<\infty$.

2.1. The formulation of the expansion

Before stating our results, we shall briefly present the formulation of the Edgeworth expansion; see Yoshida [29] for a more general exposition.

Denote by $\chi_{r,T}(u)$ the rth cumulant function of $T^{-1/2}H_T$ ($r\in\mathbb{N}$, $r\geqslant 2$), where H is defined by (2)

$$\chi_{r,T}(u) = \partial_u^r \log E[\exp(iuT^{-1/2}H_T)].$$

Define $\tilde{P}_{r,T}(u)$ by the formal expansion

$$\exp\left(\sum_{r=2}^{\infty}\frac{1}{r!}\chi_{r,T}(u)\right) = \exp\left(\frac{1}{2}\chi_{2,T}(u)\right) + \sum_{r=1}^{\infty}T^{-r/2}\tilde{P}_{r,T}(u).$$

Fix $p \in \mathbb{N}$ ($p \geqslant 3$), and define $\hat{\Psi}_{p,T}(u)$ by

$$\hat{\Psi}_{p,T}(u) = \exp\left(\frac{1}{2}\chi_{T,2}(u)\right) + \sum_{r=1}^{p-2} T^{-r/2}\tilde{P}_{r,T}(u).$$

Then the $(p-2)$th Edgeworth expansion, say $\Psi_{p,T}$, is defined by the Fourier inversion of $\hat{\Psi}_{p,T}$. Denote by $\phi(\cdot;\Sigma)$ the one-dimensional Gaussian density with mean zero and variance $\Sigma > 0$, and let $h_r(y;\Sigma)$ stand for the rth Hermite polynomial associated with $\phi(\cdot;\Sigma)$

$$h_r(y;\Sigma) = (-1)^r \phi(y;\Sigma)^{-1}\partial_y^r \phi(y;\Sigma).$$

Put $\chi_{r,T} = (-i)^r \chi_{r,T}(0)$, the rth cumulant of $T^{-1/2}H_T$, and write $\chi_{2,T}$ as Σ_T for convenience: in our case, $\chi_{r,T} = O(T^{-(r-2)/2})$ for $T \to \infty$. Then the density of $\Psi_{p,T}$ with respect to the Lebesgue measure is given by

$$g_p(y; T^{-1/2}H_T) = \{1 + G_{p,T}(y)\}\phi(y;\Sigma_T),$$

where

$$G_{p,T}(y) = \sum_{k=1}^{p-2}\sum_{l=1}^{k}\sum_{\substack{k_1,\ldots,k_l \in \mathbb{N}:\\ k_1+\cdots+k_l=k}} \frac{\chi_{k_1+2,T}\cdot\cdots\cdot\chi_{k_l+2,T}}{l!(k_1+2)!\cdots(k_l+2)!}\, h_{k+2l}(y;\Sigma_T).$$

For instance, the third-order approximation $g_4(y;T^{-1/2}H_T)$ (corresponding to the second-order Edgeworth expansion) is given by

$$g_4(y;T^{-1/2}H_T) = \phi(y;\Sigma_T)\left\{1 + \sum_{k=1}^{2} B_{k,T}(y)\right\},$$

where

$$B_{1,T}(y) = \frac{\chi_{3,T}}{3!}\left(\frac{y^3}{\Sigma_T^3} - \frac{3y}{\Sigma_T^2}\right),$$
$$B_{2,T}(y) = \frac{\chi_{4,T}}{4!}\left(\frac{y^4}{\Sigma_T^4} - \frac{6y^2}{\Sigma_T^3} + \frac{3}{\Sigma_T^2}\right) + \frac{\chi_{3,T}^2}{2!(3!)^2}\left(\frac{y^6}{\Sigma_T^6} - \frac{15y^4}{\Sigma_T^5} + \frac{45y^2}{\Sigma_T^4} - \frac{15}{\Sigma_T^3}\right).$$

Let $p_0 = 2[p/2]$ and denote by $\mathscr{E}(M,p_0)$ the set of all measurable functions $f : \mathbf{R} \to \mathbf{R}$ satisfying $|f(x)| \leqslant M(1+|x|^{p_0})$ for every $x \in \mathbb{R}$. Put

$$\Delta_{p,T}(f) = |E[f(T^{-1/2}H_T)] - \Psi_{p,T}[f]|$$

and

$$\omega(f;\delta,\nu) = \int_{\mathbb{R}} \sup_{|y|\leqslant\delta} |f(x+y) - f(x)|\nu(\mathrm{d}x)$$

for $\delta > 0$, measurable function f and Borel measure ν on $\mathbb{R}$.

Suppose that $\Sigma_T \to \Sigma > 0$ as $T \to \infty$, and fix any positive constant Σ^0 such that $\Sigma^0 > \Sigma$. We say that "*Estimate* (10) *holds true for* $T^{-1/2}H_T$" if "*for any* $M, K > 0$, there exist positive constants M^* *and* δ^* *such that*

$$\Delta_{p,T}(f) \leqslant M^* \omega(f; T^{-K}, \phi(x; \Sigma^0)\,\mathrm{d}x) + \mathrm{o}(T^{-(p-2+\delta^*)/2}) \tag{10}$$

for $T \to \infty$ *uniformly in* $f \in \mathscr{E}(M, p_0)$". Our goal is to show that Estimate (10) holds true for $T^{-1/2}H_T$. We impose the following moment condition.

Assumption 1. X is strictly stationary with invariant distribution F admitting moments of any order.

Remark 1. A natural question is that "given a desired order of the expansion, is it possible to specify up to what order of F's moments are actually required?". To answer this, apart from [A2] easy to check (see Section 4), we must carefully estimate the moment of the dominating polynomial $\mathscr{P}$ of $|\Psi(\hat{\psi}_{\varepsilon,\varepsilon'})|$ (see (36) and (37) in the proof), where the function Ψ essentially comes from the integration by parts formula, and hence the specification of the required order is in principle possible. However, we do not pursue this problem here because in most applications (cf. Barndorff-Nielsen and Shephard [5–7]), Condition [A2] is fulfilled and it is not so constructive to spare the space to count the order.

In the sequel, we denote by $\kappa_\xi^{(k)}$ the kth cumulant of ξ, a random variable or a distribution.

2.2. The result

Clearly H satisfies

$$\mathrm{d}H_t = \beta(X_t - \kappa_F^{(1)})\,\mathrm{d}t + \sqrt{X_t}\,\mathrm{d}w_t + \rho\,\mathrm{d}\bar{Z}_t, \quad H_0 = 0, \tag{11}$$

where $\bar{Z}_t = Z_t - E[Z_t] = Z_t - E[Z_1]t$ is the centred Z. Since $\lambda > 0$ and Z is a subordinator, we have $\operatorname{supp} F \subset \mathbb{R}_+$. If Π_Z admits moments of any order outside neighborhoods of the origin, then Assumption 1 is satisfied under $\mathscr{L}(X_0) = F$.

Denote by Λ_Z the Poisson random measure associated with jumps of Z, and let it be written as

$$\Lambda_Z(\mathrm{d}t, \mathrm{d}z) = \mu_Z^\flat(\mathrm{d}t, \mathrm{d}z) + \mu_Z(\mathrm{d}t, \mathrm{d}z) \tag{12}$$

for some Poisson random measures $\mu_Z^\flat$ and μ_Z. Correspondingly, write

$$\Pi_Z(\mathrm{d}z) = v_Z^\flat(\mathrm{d}z) + v_Z(\mathrm{d}z), \tag{13}$$

where $v_Z^\flat$ and v_Z stand for the Lévy measures on $\mathbb{R}_+$ associated with $\mu_Z^\flat$ and μ_Z, respectively.

Assumption 2. There exists a non-empty open subset of $\mathbb{R}_+$ on which the Lévy measure v_Z admits a positive C^3-density with respect to the Lebesgue measure.

We need the C^3-property of the density of v_Z for the condition $(\tilde{A}' - 4)$ of Bichteler et al. [8]. The Lévy measure $v_Z^\flat$ may be any one as long as Assumption 1 is satisfied; in particular, we may take $v_Z^\flat \equiv 0$ if Π_Z admits a sufficiently smooth positive density. Many examples of F treated by Barndorff-Nielsen and Shephard satisfy Assumptions 1 and 2; for instance, generalized inverse Gaussian, tempered stable and selfdecomposable modified stable (cf. [6]).

Now we are in a position to state the main result.

Theorem. *Let H be given by* (11). *Suppose that Assumptions* 1 *and* 2 *are met, and fix any positive number Σ^0 such that*

$$\Sigma^0 > \kappa_F^{(1)} + \frac{2}{\lambda}(\beta + \lambda\rho)^2 \kappa_F^{(2)}.$$

Then, Estimate (10) *holds true for $T^{-1/2} H_T$.*

The proof is deferred to Section 4.

Remark 2. It is also possible to prove Estimate (10) for (X, Y) is given by

$$\begin{aligned} \mathrm{d}X_t &= -\lambda X_t \, \mathrm{d}t + \mathrm{d}Z_t, \\ \mathrm{d}Y_t &= (\gamma + \beta X_t)\,\mathrm{d}t + \rho\,\mathrm{d}Z_t, \quad Y_0 = 0. \end{aligned}$$

The Lévy process Z here may take values in the whole line. In this case, the regularity of $\mathscr{L}(X, H)$, which plays an essential role in derivation of the expansion, is inferior to that of (1) since we have only one-dimensional random input Z against the two-dimensional objective (X, H). Hence, it is not clear whether $\mathscr{L}(X, H)$ possesses enough regularity. In particular, for pure-jump Z, this distributional problem is mathematically interesting in its own right. Under rather mild conditions, we can also guarantee the expansion even for Z of pure-jump type. It turns out that in this case the restriction $\rho\lambda + \beta \neq 0$ is necessary for non-degeneracy of the limit distribution of $T^{-1/2} H_T$. See Masuda and Yoshida [17] for details.

3. Coefficients of the expansion

As already mentioned, formula (7) is useful for computation of the coefficients of the expansion. Simple but tedious computations lead to explicit expressions of $\chi_{r,T}$.

A minor modification of Lukacs [14, Theorem 1] yields the following simple lemma.

Lemma 3. *Let Z be a subordinator, let $h : [0, T] \times \mathbb{R} \to \mathbb{C}$ be continuous in the first component, and suppose that the real part of $h(s, u)$ is non-positive for every (s, u). Then*

$$\log E\left[\exp\left\{\int_0^T h(s,u)\,\mathrm{d}Z_s\right\}\right] = \int_0^T \log E[\exp\{h(s,u)Z_1\}]\,\mathrm{d}s$$

for every $u \in \mathbb{R}$.

As in (8), it readily follows from Lemma 3 that under Assumption 1

$$\chi_{r,T}(u) = \partial_u^r \kappa(a_T(u); F) + \int_0^T \partial_u^r \kappa(b_T(v,u); Z_1)\,\mathrm{d}v, \quad r \geqslant 2, \tag{14}$$

where

$$a_T(u) = \left(\frac{u\beta}{\sqrt{T}} + i\,\frac{u^2}{2T}\right)\eta(\lambda, T),$$

$$b_T(v,u) = \frac{u}{\sqrt{T}}\,\{\beta\eta(\lambda, v) + \rho\} + i\,\frac{u^2}{2T}\,\eta(\lambda, v).$$

The elementary chain rule for differentiations and the above (14) yield the explicit expressions for $\chi_{r,T}$. Note that if $\beta = \rho = 0$, then all odd-order cumulants vanish. See Theorem 2.2 of Nicolato and Venardos [18] for the Laplace transform of $T^{-1/2}H_T$.

The following formula is convenient for computations of $\chi_{r,T}$ (for the second-term on the right-hand side in (14)):

$$J_{k,l}(T) \coloneqq \int_0^T \{\beta\eta(\lambda, v) + \rho\}^k \eta(\lambda, v)^l \,\mathrm{d}s = \sum_{j=0}^{k} \binom{k}{j} \beta^j \rho^{k-j} I_{l+j}(T) \tag{15}$$

for $k, l \in \mathbb{N} \cup \{0\}$, where

$$I_m(T) = \int_0^T \{\eta(\lambda, v)\}^m \,\mathrm{d}v, \quad m \in \mathbf{N} \cup \{0\}$$

satisfy the recurrence formula

$$I_k(T) = \lambda^{-1} I_{k-1}(T) - (\lambda k)^{-1}\{\eta(\lambda, T)\}^k, \quad k \in \mathbb{N}$$

from which we get

$$\begin{aligned} I_m(T) &= \lambda^{-m} T - \lambda^{-(m+1)} \sum_{q=1}^{m} q^{-1}\{\lambda\eta(\lambda, T)\}^q, \quad m \geqslant 1, \\ I_0(T) &= T. \end{aligned} \tag{16}$$

Eqs. (15) and (16) imply that

$$\frac{1}{T} J_{k,l}(T) \to \lambda^{-(k+l)} \sum_{j=0}^{k} \binom{k}{j} \beta^j \rho^{k-j} \lambda^{k-j} = \lambda^{-(l+k)} (\beta + \rho\lambda)^k$$

as $T \to \infty$. In particular, we get

$$\Sigma_T = \chi_{2,T} \to \kappa_F^{(1)} + \frac{2}{\lambda}\,(\beta + \lambda\rho)^2 \kappa_F^{(2)} > 0.$$

For the next two, we obtain

$$\begin{aligned} \chi_{3,T} = T^{-1/2}\{&\kappa_F^{(3)} T^{-1}(\beta^3(\eta(\lambda, T))^3 + 3\lambda J_{3,0}(T)) \\ &+ 3\kappa_F^{(2)} T^{-1}(\beta(\eta(\lambda, T))^2 + 2\lambda J_{1,1}(T))\} \\ &\sim T^{-1/2}\{3\lambda^{-2}(\beta + \rho\lambda)^3 \kappa_F^{(3)} + 6\lambda^{-1}(\beta + \rho\lambda)\kappa_F^{(2)}\} \end{aligned}$$

and

$$\begin{aligned}\chi_{4,T} &= T^{-1}\{\kappa_F^{(4)}T^{-1}(\beta^4(\eta(\lambda,T))^4 + 4\lambda J_{4,0}(T)) \\ &\quad + 6\kappa_F^{(3)}T^{-1}(\beta^2(\eta(\lambda,T))^3 + 3\lambda J_{2,1}(T)) \\ &\quad + 3\kappa_F^{(2)}T^{-1}((\eta(\lambda,T))^2 + 2\lambda I_2(T))\} \\ &\sim T^{-1}\{4\lambda^{-3}(\beta+\rho\lambda)^4\kappa_F^{(4)} + 18\lambda^{-2}(\beta+\rho\lambda)^2\kappa_F^{(3)} + 6\lambda^{-1}\kappa_F^{(2)}\},\end{aligned}$$

where $F_T \sim G_T$ means that $F_T/G_T \to 1$ as $T \to \infty$. As is mentioned, $\chi_{r,T} = \mathrm{O}(T^{-(r-2)/2})$ in general.

Remark 4. Barndorff-Nielsen and Shephard [6] advocated that the tempered stable distribution denoted by $TS(\kappa,\delta,\xi)$, where $0<\kappa<1$, $\delta>0$, and $\xi \geqslant 0$, is one of good candidates for F when the model is applied to finance; a special case is $IG(\delta,\xi)$ for $\kappa = \frac{1}{2}$. For $TS(\kappa,\delta,\xi)$, we must assume that $\xi>0$ for Assumption 1, and in this case the normal tempered stable distribution (NTS) including the normal inverse Gaussian (NIG) for $\kappa = \frac{1}{2}$ appears as the approximation of the distribution of the instantaneous log-return. NTS as well as NIG is known to be able to exhibit skewness and steepness (fat tails) very flexibly and it also possesses the reproducing-property. Further, the cumulant generating function of $\mathrm{TS}(\kappa,\delta,\xi)$ is simply given by $\delta\{\xi - (\xi^{1/\kappa} - 2u)^\kappa\}$, from which one can easily get

$$\kappa^{(k)}_{TS(\kappa,\delta,\xi)} = -\delta(-2)^k \xi^{(\kappa-k)/\kappa} \prod_{j=0}^{k-1}(\kappa - j), \quad k \in \mathbb{N}.$$

4. Proof of theorem

The proof will be carried out essentially by applying Theorem 4 of Yoshida [29], which targets at stochastic differential equations with jumps. The theorem just referred to is a special case of Theorem 1 of Yoshida [29] covering general partial mixing processes, hence, for reference let us briefly mention Theorem 1 of Yoshida [29] before entering the proof.

Building on the Markov nature and stationarity of X, the exponential mixing version of Theorem 1 of Yoshida [29] asserts that it suffices to verify the following conditions:

[A1] X is strongly mixing with exponential rate;
[A2] for each $T \in \mathbb{R}_+$, $\sup_{t\in[0,T]} \|H_t\|_{L^{p+1}(P)} < \infty$;
[A3] (*a version of conditional type Cramér conditions*) there exist positive constants t^0, a, a' and B, and a truncation functional $\psi : (\Omega, \mathscr{F}) \to ([0,1], \mathscr{B}([0,1]))$ such that $0<a, a'<1$, $4a'<(a-1)^2$ and that the following two conditions are met:

$$E\left[\sup_{|u|\geqslant B} |E[\psi \mathrm{e}^{\mathrm{i}uH_{t^0}} | X_0, X_{t^0}]|\right] < a', \tag{17}$$

$$1 - E[\psi] < a. \tag{18}$$

It is difficult in general to check [A3] directly, however, we can employ infinite dimensional stochastic calculus (Malliavin calculus) with truncation to verify it, and resultingly, more easy-to-check conditions than those of Theorem 1 of Yoshida [29] can be given: this is just what Theorem 4 of Yoshida [29] provides. There [A3] is replaced by another condition called $[A3^{Q}]$, in which local non-degeneracy of a Malliavin covariance matrix of interest as well as some other regularity conditions is required. Our plan is thus to verify [A1], [A2] and $[A3^{Q}]$ under our assumptions.

We see that Assumption 1 directly ensures [A1] and [A2]: X is exponentially β-mixing hence exponentially strong-mixing under Assumption 1, see Masuda [15, Theorem 4.3] for details; turning to [A2], the relation (6) implies that Z_1 as well as F admits moments of any order, hence Burkholder-Davis-Gundy's and Jensen's inequalities readily ensure [A2]. Thus it remains to verify [A3].

In addition to direct application of Theorem 4 of Yoshida [29] itself, we shall introduce an auxiliary process $\tilde{H}$ for H, which will turn out to be essential for the condition $(\tilde{A}' - 4)$ of Bichteler et al. [8] to be fulfilled in our context. Here the condition $(\tilde{A}' - r)$, $r \in \mathbb{N}$, is a series of conditions for smoothness of the coefficients of stochastic differential equations of interest, moreover, it requires polynomial growth rate of the derivatives of the coefficients; see p. 147 of Bichteler et al. [8] for details. More precisely, we shall circumvent the irregular behavior of the derivatives of H's diffusion coefficient $\sqrt{X_t}$ near the origin, introducing a suitable truncation functional.

In the rest of this section, we write $\tilde{\mu}_Z^{\flat}(\mathrm{d}t, \mathrm{d}z) = \mu_Z^{\flat}(\mathrm{d}t, \mathrm{d}z) - \nu_Z^{\flat}(\mathrm{d}z)\,\mathrm{d}t$ and $\tilde{\mu}_Z(\mathrm{d}t, \mathrm{d}z) = \mu_Z(\mathrm{d}t, \mathrm{d}z) - \nu_Z(\mathrm{d}z)\,\mathrm{d}t$; recall (12) and (13).

4.1. *Transforming the Poisson random measure*

Under Assumption 1, the Lévy–Itô decomposition gives

$$Z_t = \lambda \kappa_F^{(1)} t + \int_0^t \int_{\mathbb{R}_+} z \tilde{\mu}_Z^{\flat}(\mathrm{d}s, \mathrm{d}z) + \int_0^t \int_{\mathbb{R}_+} z \tilde{\mu}_Z(\mathrm{d}s, \mathrm{d}z)$$

for each $t \in \mathbb{R}_+$. Under Assumption 2, we can find an open set $E_{A,0} = (c_1, c_2)$ with $0 < c_1 < c_2 < \infty$, on which ν_Z admits a C^3-density g_Z such that $\inf_{z \in E_{A,0}} g_Z(z) > 0$.

To begin with, we partly rewrite the stochastic differential equation of (X, H), replacing partial jumps associated with μ_Z corresponding to the region (c_1, c_2) by the uniform Poisson space, so that the resulting compensating measure becomes the Lebesgue measure; this is required for direct application of the theory of Bichteler et al. [8]. Under Assumption 2, this corresponds to the change of variable

$$z^* = z^*(z) = \int_z^{c_2} g_Z(v)\,\mathrm{d}v, \quad z \in E_{A,0}. \tag{19}$$

Write $g_Z^+(z) = z^*(z)$. Then $g_Z^+(z)$ is strictly decreasing on $E_{A,0}$, hence $g_Z^+(c_1) > g_Z^+(c_2) > 0$. Accordingly we have

$$\int_0^t \int_{c_1}^{c_2} z \tilde{\mu}_Z(\mathrm{d}s, \mathrm{d}z) = \int_0^t \int_{g_Z^+(c_2)}^{g_Z^+(c_1)} g_Z^-(z^*) \tilde{\mu}_Z^*(\mathrm{d}s, \mathrm{d}z^*), \tag{20}$$

where g_Z^- stands for the inverse function of $z \mapsto g_Z^+(z)$, which is also strictly decreasing, and $\tilde{\mu}_Z^*(\mathrm{d}t, \mathrm{d}z^*) = \mu_Z^*(\mathrm{d}t, \mathrm{d}z^*) - \mathrm{d}t\,\mathrm{d}z^*$ with the integer-valued random measure μ_Z^* defined by

$$\int_0^t \int_{a_1}^{a_2} h(s,z)\mu_Z(\mathrm{d}s, \mathrm{d}z) = \int_0^t \int_{g_Z^+(a_2)}^{g_Z^+(a_1)} h(s, g_Z^-(z^*))\mu_Z^*(\mathrm{d}s, \mathrm{d}z^*) \tag{21}$$

for each $t \in \mathbb{R}_+$, $a_1, a_2 \in \mathbb{R}$ such that $a_1 < a_2$, and for any measurable function h on $\mathbb{R}_+ \times \mathbb{R}_+$. Put $E_A = (g_Z^+(c_2), g_Z^+(c_1))$. For $B \in \mathscr{B}(E_A)$ and $t \in \mathbb{R}_+$ we have

$$E[\mu_Z^*([0,t], B)] = l(B)t,$$

where $l(\cdot)$ stands for the Lebesgue measure. Then the stochastic differential equation of (X, H) becomes

$$\begin{aligned} \begin{pmatrix} \mathrm{d}X_t \\ \mathrm{d}H_t \end{pmatrix} &= (\kappa_F^{(1)} - X_t)\begin{pmatrix} \lambda \\ -\beta \end{pmatrix}\mathrm{d}t + \begin{pmatrix} 0 \\ \sqrt{X_t} \end{pmatrix}\mathrm{d}w_t \\ &\quad + \int_{\mathbb{R}_+} z\begin{pmatrix} 1 \\ \rho \end{pmatrix}\{\tilde{\mu}_Z^\flat + 1_{E_{A,0}^c}\tilde{\mu}_Z\}(\mathrm{d}t, \mathrm{d}z) + \int_{\tilde{E}_A} J_A(z^*)\begin{pmatrix} 1 \\ \rho \end{pmatrix}\tilde{\mu}_Z^*(\mathrm{d}t, \mathrm{d}z^*), \end{aligned} \tag{22}$$

where $E_{A,0}^c$ denotes the complement of $E_{A,0}$, $\tilde{E}_A = E_A \cup (g_Z^+(c_1), \infty)$, and

$$J_A(z^*) = g_Z^-(z^*)1_{E_A}(z^*), \quad z^* \in \tilde{E}_A.$$

Note that (22) is clearly graded associated with the grading $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ of $\mathbb{R}^2$ in the sense of 5-5 of Bichteler et al. [8]. Also, note that for each $t \in \mathbb{R}_+$ the random number $\mu_Z^*([0,t], E_A)$ is a.s. finite, and that the function $z^* \mapsto J_A(z^*)$ is of class C^4 on $\tilde{E}_A$ by virtue of Assumption 2 and the inverse function theorem.

Remark 5. We have presented a (partial) transformation of the Poisson random measure μ_Z demonstratively, however, we should note that it is always possible to extract a uniform Poisson random measure from any Poisson random measure μ on $I \times E \subset \mathbb{R}_+ \times \mathbb{R}$, at least as soon as μ's Lévy measure admits a positive density on E. Of course this is true of the multi-dimensional case.

Let $(\hat{\Omega}, \hat{\mathscr{G}}, \hat{P})$ be the canonical space defined as follows. Let $(\tilde{\Omega}, \tilde{\mathscr{G}}, \tilde{P})$ stand for the canonical product Wiener–Poisson space over a non-empty time-interval $[0, t^0]$, and then define $(\hat{\Omega}, \hat{\mathscr{G}})$ by the product measurable space $(\hat{\Omega}, \hat{\mathscr{G}}) = (\mathbb{R}_+ \times \tilde{\Omega}, \mathscr{B}(\mathbb{R}_+) \otimes \tilde{\mathscr{G}})$. Define a probability measure $\hat{P}$ by $\hat{P} = F \times \tilde{P}$: under $\hat{P}$, the projection to the first space, say $\hat{x}$, yields the same law as F, the canonical projection w is a one-dimensional Wiener process, and the canonical projections $\mu_Z^\flat + 1_{E_{A,0}^c}\mu_Z$ and μ_Z^* are independent Poisson random measures on $[0, t^0] \times \mathbb{R}_+$ and $[0, t^0] \times \tilde{E}_A$, respectively. Also, $\hat{x}$ and $(w, \mu_Z^\flat + 1_{E_{A,0}^c}\mu_Z, \mu_Z^*)$ are independent under $\hat{P}$. We shall consistently write Z for its distributional equivalent on the space $(\hat{\Omega}, \hat{\mathscr{G}}, \hat{P})$, i.e. $\mathscr{L}(Z|P) = \mathscr{L}(Z|\hat{P})$, where $\mathscr{L}(\xi|Q)$ stands for the distribution of a random variable ξ under a probability measure Q: accordingly, we still write $\bar{Z}_t = Z_t - \hat{E}[Z_1]t$.

On the space $(\hat{\Omega}, \hat{\mathscr{G}}, \hat{P})$, we consider the flow $(X(t,v), H(t,v))^\top$ associated with (X, H) starting from $v = (x,h)^\top \in \mathbb{R}_+ \times \mathbb{R}$

$$\begin{aligned} X(t,v) &= \mathrm{e}^{-\lambda t} x + \int_0^t \mathrm{e}^{-\lambda(t-s)}\, \mathrm{d}Z_s, \\ H(t,v) &= h + \beta \int_0^t (X(s,v) - \kappa_F^{(1)})\, \mathrm{d}s + \int_0^t \sqrt{X(s,v)}\, \mathrm{d}w_s + \rho \bar{Z}_t. \end{aligned} \tag{23}$$

We shall execute the Malliavin calculus for this flow on a suitable event $\{\hat{\psi}_{\varepsilon,\varepsilon'} > 0\}$, where $\hat{\psi}_{\varepsilon,\varepsilon'}$ is the truncation functional introduced in the next subsection.

4.2. Construction of a truncation functional

Here we concretely construct a truncation functional $\hat{\psi}_{\varepsilon,\varepsilon'}$ defined on $(\hat{\Omega}, \hat{\mathscr{G}}, \hat{P})$, in order to extract a "nice event" on which an integration-by-parts formula can be applied: the meaning of the argument $(\varepsilon, \varepsilon')$ will be clarified below. We must show that such an event has positive $\hat{P}$-probability. The functional $\hat{\psi}_{\varepsilon,\varepsilon'}$ corresponds to a distributional equivalent of ψ appearing in [A3].

Let $\varphi_1 \in C_B^\infty(\mathbb{R}_+; [0,1])$ be a non-increasing function such that $\varphi_1(x) = 1$ if $0 \leqslant x \leqslant 1/2$ and $\varphi_1(x) = 0$ if $x \geqslant 1$, where $C_B^\infty(\mathbb{R}_+; [0,1])$ denotes the set of all $[0,1]$-valued smooth functions defined on $\mathbb{R}_+$ with bounded derivatives. We shall consider $\hat{\psi}_{\varepsilon,\varepsilon'}$ of the form

$$\hat{\psi}_{\varepsilon,\varepsilon'} = \varphi_1(\hat{\xi}_{\varepsilon,\varepsilon'}) \tag{24}$$

for some $\hat{\xi}_{\varepsilon,\varepsilon'} \in D^L_{2,\infty-}$, where $D^L_{2,\infty-}$ denotes the domain of the extended Malliavin operator L employed in Yoshida [29, Section 5]: see Bichteler et al. [8, Section 9] for a detailed exposition.

From now on, we shall construct a "nice event" step by step, and then suitably define $\hat{\xi}_{\varepsilon,\varepsilon'}$ ((33) below). In what follows, we fix arbitrary positive constants x_0 and t^0, and put $\hat{v} = (\hat{x}, 0)^\top$.

Step 1. Define an auxiliary event $\mathscr{A}_1$ by

$$\mathscr{A}_1 = \{\hat{x} \geqslant \mathrm{e}^{\lambda t^0} x_0\}.$$

Clearly $\hat{P}[\mathscr{A}_1] > 0$ since any non-trivial selfdecomposable distribution possesses an unbounded support. Since Z is a subordinator, from (23) we see that $X(t, \hat{v}) \geqslant \mathrm{e}^{-\lambda t}\hat{x} \geqslant \mathrm{e}^{\lambda(t^0 - t)} x_0 \geqslant x_0$ on $\mathscr{A}_1$ for every $t \in [0, t^0]$, so that we have $\inf_{0 \leqslant t \leqslant t^0} X(t, \hat{v}) \geqslant x_0$ uniformly on $\mathscr{A}_1$.

Fix any function $\tau \in C_b^\infty(\mathbb{R}_+; \mathbb{R}_+)$ satisfying the following conditions, where $C_b^\infty(\mathbb{R}_+; \mathbb{R}_+)$ stands for the set of all smooth functions on $\mathbb{R}_+$ with bounded derivatives of order $\geqslant 1$:

(τ-1) $\tau(x) = \sqrt{x}$ for $x \geqslant x_0/7$;
(τ-2) $x \mapsto \tau(x)$ and $x \mapsto \partial\tau(x)$ are globally Lipschitz.

Using this τ, define a process $\tilde{H}(\cdot, v)$ by

$$\tilde{H}(t, v) = h + \beta \int_0^t (X(s,x) - \kappa_F^{(1)})\, \mathrm{d}s + \int_0^t \tau(X(s,x))\, \mathrm{d}w_s + \rho \bar{Z}_t$$

which is same as H except for the smooth diffusion coefficient. By the previous paragraph, $H(\cdot, v) = \tilde{H}(\cdot, v)$ for $t \in [0, t^0]$ on $\mathscr{A}_1$.

Step 2. Let c'_j and c''_j ($j = 1, 2$) be positive constants such that $0 < c_1 < c'_1 < c''_1 < c''_2 < c'_2 < c_2 < \infty$, and write $\check{E}_A = (g_Z^+(c''_2), g_Z^+(c''_1)) \Subset E_A$. Let $\eta_A \in C_B^\infty(\mathbb{R}_+; \mathbb{R}_+)$ be any function satisfying $\inf_{z^* \in \check{E}_A} \eta_A(z^*) > 0$, and $\eta_A(z^*) = 0$ for $z^* \notin (g_Z^+(c'_2), g_Z^+(c'_1))$: we shall utilize this η_A as an auxiliary function satisfying 10-1 of Bichteler et al. [8].

Denote by ∇ the differential operator with respect to $v = (x, h)^\top$. On account of the expression (23), the matrix-valued process $\tilde{K}(\cdot, v) = \nabla(X(\cdot, v), \tilde{H}(\cdot, v))^\top$ is given by

$$\tilde{K}(t, v) = \begin{pmatrix} \mathrm{e}^{-\lambda t} & 0 \\ \beta\lambda^{-1}(1 - \mathrm{e}^{-\lambda t}) + \partial_x \int_0^t \tau(X(s, v))\, \mathrm{d}w_s & 1 \end{pmatrix}. \tag{25}$$

Denote by A_t the (2,1)-component of the right-hand side of (25). In view of Assumption 1, the definition of τ, and (23), it is clear that $E[\int_0^{t^0} \tau(X(s, \hat{v}))^2\, \mathrm{d}s] < \infty$ and $E[\int_0^{t^0} \{\partial_x \tau(X(s, \hat{v}))\}^2\, \mathrm{d}s] < \infty$. Then it is well known that the Lipschitz property (τ-2) ensures existence of a differentiable version of $x \mapsto \int_0^t \tau(X(s, v))\, \mathrm{d}w_s$, so we have

$$\tilde{A}_t := \partial_x \int_0^t \tau(X(s, v))\, \mathrm{d}w_s \bigg|_{x = \hat{x}} = \int_0^t \mathrm{e}^{-\lambda s}(\partial\tau) \circ (X(s, v))\, \mathrm{d}w_s \bigg|_{x = \hat{x}}.$$

Fix $t_1 \in (0, t^0)$ and $z_0 \in \check{E}_A$. Take a sufficiently small constant $\varepsilon > 0$ so that $I_1^\varepsilon := (t_1 - \varepsilon, t_1 + \varepsilon) \subset (0, t^0)$ and that $E_A^\varepsilon := (z_0 - \varepsilon, z_0 + \varepsilon) \subset \check{E}_A$. Now we define $\mathscr{A}_2^\varepsilon$ by

$$\mathscr{A}_2^\varepsilon = \{\mu_Z^*(I_1^\varepsilon, E_A^\varepsilon) = 1\}. \tag{26}$$

Obviously $\hat{P}[\mathscr{A}_2^\varepsilon] = 4\varepsilon^2 \exp(-4\varepsilon^2) > 0$ for any $\varepsilon > 0$.

Step 3. Next, for $\varepsilon' > 0$ we introduce

$$\mathscr{A}_3^{\varepsilon'} = \left\{ \sup_{0 \leqslant t \leqslant t^0} |\tilde{A}_t| < \varepsilon' \right\}. \tag{27}$$

Because of the boundedness of $x \mapsto \partial\tau(x)$, $\tilde{A}$ is a continuous $\mathbf{F}$-martingale. Enlarging the underlying stochastic basis, we see that there exists a standard Wiener process $B = (B_t)_{t \in \mathbb{R}_+}$ such that $\tilde{A}_t = B_{[\tilde{A}]_t}$ (e.g. Rogers and Williams [22, Theorem IV 34.11]), where $[\tilde{A}]_t = \int_0^t \mathrm{e}^{-2\lambda s}\{(\partial\tau) \circ (X(s, \hat{v}))\}^2\, \mathrm{d}s$, and obviously $[\tilde{A}]_{t^0} \leqslant \|\partial\tau\|_\infty^2 t^0$. Therefore, we can estimate as

$$\begin{aligned} \hat{P}[\mathscr{A}_3^{\varepsilon'} | \mathscr{A}_2^\varepsilon] &= \hat{P}\left[\hat{P}\left[\sup_{0 \leqslant t \leqslant t^0} |B_{[\tilde{A}]_t}| < \varepsilon' \middle| \sigma(X, \mu_Z^*) \right] \middle| \mathscr{A}_2^\varepsilon \right] \\ &\geqslant \hat{P}\left[\hat{P}\left[\sup_{0 \leqslant t \leqslant \|\partial\tau\|_\infty^2 t^0} |B_t| < \varepsilon' \middle| \sigma(X, \mu_Z^*) \right] \middle| \mathscr{A}_2^\varepsilon \right], \end{aligned}$$

where the random number

$$\hat{P}\left[\sup_{0\leqslant t\leqslant \|\partial\tau\|_\infty^2 t^0} |B_t| < \varepsilon' \middle| \sigma(X,\mu_Z^*)\right]$$

is a.s. positive for any $t^0, \varepsilon' > 0$ (cf. Billingsley [9, p. 97]). Hence we obtain that $\hat{P}[\mathscr{A}_3^{\varepsilon'}|\mathscr{A}_2^{\varepsilon}] > 0$ a.s. Putting $\mathscr{A}^{\varepsilon,\varepsilon'} = \mathscr{A}_1 \cap \mathscr{A}_2^{\varepsilon} \cap \mathscr{A}_3^{\varepsilon'}$, we have

$$\begin{aligned}\hat{P}[\mathscr{A}^{\varepsilon,\varepsilon'}] &= \hat{P}[\mathscr{A}_1]\hat{P}[\mathscr{A}_2^{\varepsilon} \cap \mathscr{A}_3^{\varepsilon'}] \\ &= \hat{P}[\mathscr{A}_1]\hat{P}[\mathscr{A}_2^{\varepsilon}]\hat{P}[\mathscr{A}_3^{\varepsilon'}|\mathscr{A}_2^{\varepsilon}] \\ &> 0\end{aligned}$$

for any ε and ε'. Note that we can control ε and ε' independently due to the independence between μ_Z^* and w.

Step 4. With the smooth modification $\tilde{H}$ introduced before, the Malliavin covariance matrix $U(\cdot,\hat{v})$ associated with the flow $(X(\cdot,\hat{v}),\tilde{H}(\cdot,\hat{v}))^\top$ is well-defined for $t \in [0,t^0]$, and given by

$$U(t,\hat{v}) = \tilde{K}(t,\hat{v})\tilde{S}(t,\hat{v})\tilde{K}(t,\hat{v})^\top, \quad t \in [0,t^0], \tag{28}$$

where, on $\mathscr{A}^{\varepsilon,\varepsilon'}$,

$$\begin{aligned}\tilde{S}(t,\hat{v}) = {} & \int_0^t \tilde{K}(s,\hat{v})^{-1}\begin{pmatrix} 0 & 0 \\ 0 & X(s,\hat{v})\end{pmatrix}\tilde{K}(s,\hat{v})^{\top\ -1}\,\mathrm{d}s \\ & + \int_0^t\int_{E_A} V_A(z^*)\tilde{K}(s,\hat{v})^{-1}\begin{pmatrix} 1 & \rho \\ \rho & \rho^2\end{pmatrix}\tilde{K}(s,\hat{v})^{\top\ -1}\mu_Z^*(\mathrm{d}s,\mathrm{d}z^*)\end{aligned} \tag{29}$$

with $V_A(z^*) = \{\partial J_A(z^*)\}^2\eta_A(z^*)$: see Bichteler et al. [8, Section 10] for details. Due to (25) and non-negative definiteness of the second term of the right-hand side of (29), we see that

$$\begin{aligned}\tilde{S}(t^0,\hat{v}) \geqslant {} & \int_0^{t^0} \tilde{K}(s,\hat{v})^{-1}\begin{pmatrix} 0 & 0 \\ 0 & X(s,\hat{v})\end{pmatrix}\tilde{K}(s,\hat{v})^{\top\ -1}\,\mathrm{d}s \\ & + \int_{I_1^\varepsilon}\int_{E_A^\varepsilon} V_A(z^*)\tilde{K}(s,\hat{v})^{-1}\begin{pmatrix} 1 & \rho \\ \rho & \rho^2\end{pmatrix}\tilde{K}(s,\hat{v})^{\top\ -1}\mu_Z^*(\mathrm{d}s,\mathrm{d}z^*) \\ = {} & \begin{pmatrix} \int_{I_1^\varepsilon}\int_{E_A^\varepsilon} V_A(z^*)\mathrm{e}^{2\lambda s}\mu_Z^*(\mathrm{d}s,\mathrm{d}z^*) & \text{sym.} \\ \int_{I_1^\varepsilon}\int_{E_A^\varepsilon} V_A(z^*)\mathrm{e}^{\lambda s}(\rho - \mathrm{e}^{\lambda s}A_s)\mu_Z^*(\mathrm{d}s,\mathrm{d}z^*) & \int_{I_1^\varepsilon}\int_{E_A^\varepsilon} V_A(z^*)(\rho - \mathrm{e}^{\lambda s}A_s)^2\mu_Z^*(\mathrm{d}s,\mathrm{d}z^*) + \int_0^{t^0} X(s,\hat{v})\,\mathrm{d}s \end{pmatrix}\end{aligned}$$

on $\mathscr{A}^{\varepsilon,\varepsilon'}$, hence

$$\begin{aligned}\det \tilde{S}(t^0,\hat{v}) \geqslant & \left(\int_{I_1^\varepsilon}\int_{E_A^\varepsilon} V_A(z^*)\mathrm{e}^{2\lambda s}\mu_Z^*(\mathrm{d}s,\mathrm{d}z^*)\right) \\ & \times\left(\int_{I_1^\varepsilon}\int_{E_A^\varepsilon} V_A(z^*)(\rho-\mathrm{e}^{\lambda s}A_s)^2\mu_Z^*(\mathrm{d}s,\mathrm{d}z^*)+\int_0^{t^0} X(s,\hat{v})\,\mathrm{d}s\right) \\ & -\left(\int_{I_1^\varepsilon}\int_{E_A^\varepsilon} V_A(z^*)\mathrm{e}^{\lambda s}(\rho-\mathrm{e}^{\lambda s}A_s)\mu_Z^*(\mathrm{d}s,\mathrm{d}z^*)\right)^2. \end{aligned} \tag{30}$$

Clearly

$$\det U(t^0,\hat{v}) = \mathrm{e}^{-2\lambda t^0}\det \tilde{S}(t^0,\hat{v}) \tag{31}$$

in view of (25) and (28). We shall show that $\det \tilde{S}(t^0,\hat{v})>0$ to conclude that $\det U(t^0,\hat{v})>0$ uniformly on $\mathscr{A}^{\varepsilon,\varepsilon'}$, i.e. local non-degeneracy of $U(t^0,\hat{v})$. In the sequel, we use the small order symbol $\mathrm{o}''(1)$ for random or non-random variables $R_{\varepsilon,\varepsilon'}$ such that $R_{\varepsilon,\varepsilon'}\to 0$ as $\varepsilon,\varepsilon'\downarrow 0$ uniformly on $\mathscr{A}^{\varepsilon,\varepsilon'}$.

Under Assumption 2, $z^*\mapsto V_A(z^*)$ is of class C^3 and strictly positive uniformly on E_A^ε. Apply Taylor's theorem around z_0 and t_1 to obtain

$$\begin{aligned} & V_A(z^*)\mathrm{e}^{2\lambda s} = V_A(z_0)\mathrm{e}^{2\lambda t_1}+\mathrm{o}''(1), \\ & V_A(z^*)(\rho-\mathrm{e}^{\lambda s}A_s)^2 = V_A(z_0)\{\rho-\beta\lambda^{-1}(\mathrm{e}^{\lambda t_1}-1)\}^2+\mathrm{o}''(1), \\ & V_A(z^*)\mathrm{e}^{\lambda s}(\rho-\mathrm{e}^{\lambda s}A_s) = V_A(z_0)\mathrm{e}^{\lambda t_1}\{\rho-\beta\lambda^{-1}(\mathrm{e}^{\lambda t_1}-1)\}+\mathrm{o}''(1). \end{aligned}$$

Substituting these three displays in (30), we get

$$\begin{aligned}\det \tilde{S}(t^0,\hat{v}) \geqslant & \{V_A(z_0)\mathrm{e}^{2\lambda t_1}+\mathrm{o}''(1)\} \\ & \times\left\{V_A(z_0)(\rho+\lambda^{-1}\beta-\lambda^{-1}\beta\mathrm{e}^{\lambda t_1})^2+\int_0^{t^0} X(s,\hat{v})\,\mathrm{d}s+\mathrm{o}''(1)\right\} \\ & -\{V_A(z_0)\mathrm{e}^{\lambda t_1}(\rho+\lambda^{-1}\beta-\lambda^{-1}\beta\mathrm{e}^{\lambda t_1})+\mathrm{o}''(1)\}^2 \\ = & V_A(z_0)\mathrm{e}^{2\lambda t_1}\int_0^{t^0} X(s,\hat{v})\,\mathrm{d}s+\mathrm{o}''(1). \end{aligned} \tag{32}$$

Here we used the fact that, for any $\varepsilon,\varepsilon'>0$, $\mu_Z^*(I_1^\varepsilon,E_A^\varepsilon)=1$ on $\mathscr{A}^{\varepsilon,\varepsilon'}$. Therefore it follows from (31) and (32) that

$$\begin{aligned}\det U(t^0,\hat{v}) &= \mathrm{e}^{-2\lambda t^0}\det \tilde{S}(t^0,\hat{v}) \\ &\geqslant \mathrm{e}^{2\lambda(t_1-t^0)}V_A(z_0)x_0t^0+\mathrm{o}''(1)\end{aligned}$$

on $\mathscr{A}^{\varepsilon,\varepsilon'}$. Without loss of generality we may suppose that $\eta_A(z_0)$ is sufficiently large (by choosing η_A suitably), so letting ε and ε' be sufficiently small we may take

$$\det U(t^0,\hat{v})\geqslant 3$$

on $\mathscr{A}^{\varepsilon,\varepsilon'}$. Fix ε and ε' like this in the rest of the proof.

Step 5. Now we define a functional $\hat{\xi}_{\varepsilon,\varepsilon'} \in D^L_{2,\infty-}$ by

$$\hat{\xi}_{\varepsilon,\varepsilon'} = \frac{1}{1+\det U(t^0,\hat{v})} + \frac{2}{1+7\hat{x}x_0^{-1}\mathrm{e}^{-\lambda t^0}}; \tag{33}$$

it is clear that $\hat{\xi}_{\varepsilon,\varepsilon'} \in \bigcap_{p<\infty} L^p(\hat{P})$. By the choice of ε and ε' in the previous step, we see that

$$\begin{aligned}
0 < \hat{P}[\mathscr{A}^{\varepsilon,\varepsilon'}] &\leqslant \hat{P}[\det U(t^0,\hat{v}) \geqslant 3,\ \hat{x} \geqslant \mathrm{e}^{\lambda t^0} x_0] \\
&\leqslant \hat{P}\left[\frac{1}{1+\det U(t^0,\hat{v})} \leqslant \frac{1}{4},\quad \frac{2}{1+7\hat{x}x_0^{-1}\mathrm{e}^{-\lambda t^0}} \leqslant \frac{1}{4}\right] \\
&\leqslant \hat{P}\left[\hat{\xi}_{\varepsilon,\varepsilon'} \leqslant \frac{1}{2}\right].
\end{aligned}$$

Consequently, $\det U(t^0,\hat{v}) = 0$ implies $\hat{\psi}_{\varepsilon,\varepsilon'}\{\det U(t^0,\hat{v})\}^{-1} = 0$ (with the convention $0 \cdot \infty = 0$). We thus end up with

Lemma 6. *Let $\hat{\psi}_{\varepsilon,\varepsilon'}$ be of the form* (24). *Then there exists $\hat{\xi}_{\varepsilon,\varepsilon'} \in D^L_{2,\infty-}$ such that $\hat{P}[\hat{\xi}_{\varepsilon,\varepsilon'} \leqslant \frac{1}{2}] > 0$ and that $\hat{\psi}_{\varepsilon,\varepsilon'}\{\det U(t^0,\hat{v})\}^{-1} \in \bigcap_{p<\infty} L^p(\hat{P})$ for each $t^0 > 0$.*

4.2.1. On the condition ($\tilde{A}' - 4$)

We must check ($\tilde{A}' - 4$) of Bichteler et al. [8] for the flow $(X(\cdot,v), H(\cdot,v))^\top$. Here ε and ε' are fixed so that the assertion of Lemma 6 holds true.

As already mentioned, the diffusion coefficient $\sqrt{X(t,v)}$ of $H(\cdot,v)$ causes trouble for ($\tilde{A}' - 4$). However, *it is sufficient that we can apply the integration-by-parts formula on the event carved out by the truncation functional $\hat{\psi}_{\varepsilon,\varepsilon'}$*. Now, let us note that the definition (24) leads to the following inclusive relation:

$$\begin{aligned}
\{\hat{\psi}_{\varepsilon,\varepsilon'} > 0\} &\subset \{\hat{\xi}_{\varepsilon,\varepsilon'} \leqslant 1\} \\
&\subset \left\{\frac{2}{1+7\hat{x}x_0^{-1}\mathrm{e}^{-\lambda t^0}} \leqslant 1\right\} \\
&= \left\{\frac{x_0\mathrm{e}^{\lambda t^0}}{7} \leqslant \hat{x}\right\} \\
&\subset \left\{\inf_{0\leqslant s\leqslant t^0} X(s,\hat{v}) \geqslant \frac{x_0}{7}\right\}.
\end{aligned}$$

Thus, the property (τ-1) implies that $H(t,\hat{v}) = \tilde{H}(t,\hat{v})$ for $t \in [0,t^0]$ on $\{\hat{\psi}_{\varepsilon,\varepsilon'} > 0\}$: in other words, we have

$$\mathscr{L}\{(\psi_{\varepsilon,\varepsilon'}, 1_{\{\psi_{\varepsilon,\varepsilon'}>0\}}(X_{t^0}, H_{t^0}))|P\} = \mathscr{L}\{(\hat{\psi}_{\varepsilon,\varepsilon'}, 1_{\{\hat{\psi}_{\varepsilon,\varepsilon'}>0\}}(X(t^0,\hat{v}), H(t^0,\hat{v})))|\hat{P}\},$$

where $\psi_{\varepsilon,\varepsilon'}$ and H_{t^0} (both defined on the original probability space $(\Omega,\mathscr{F},P)$) stand for a distributional equivalent of $\hat{\psi}_{\varepsilon,\varepsilon'}$ and $\tilde{H}(t^0,0)$, respectively. On the other hand, it is quite straightforward to verify ($\tilde{A}' - 4$) for $\{(X(\cdot,\hat{v}), \tilde{H}(\cdot,\hat{v}))^\top\}_{t\in[0,t^0]}$, so that we have obtained

Lemma 7. *Under Assumptions* 1 *and* 2, *the process* $\{1_{\{\hat{\psi}_{\varepsilon,\varepsilon'}>0\}}(X(\cdot,\hat{v}),H(\cdot,\hat{v}))^\top\}_{t\in[0,t^0]}$ *meets* $(\tilde{A}'-4)$.

4.2.2. An integration-by-parts formula and moment conditions

Here ε and ε' are still fixed as the assertion of Lemma 6 holds true. On $(\hat{\Omega},\hat{\mathscr{G}},\hat{P})$, consider the Malliavin operator $(L,D^L_{2,\infty-})$. Denote by Γ_L the bilinear form corresponding to L: namely, for $F,G\in D^L_{2,\infty-}$

$$\Gamma_L(F,G)=L(FG)-GLF-FLG. \tag{34}$$

Put $\hat{\mathscr{X}}=(X(t^0,\hat{v}),H(t^0,\hat{v}))$ and $S_1^*[\hat{\psi}_{\varepsilon,\varepsilon'};\hat{\mathscr{X}}]=\{\sigma^{pq}_{\hat{\mathscr{X}}},\Delta^{-1}_{\hat{\mathscr{X}}}\hat{\psi}_{\varepsilon,\varepsilon'}\}$, where $\sigma_{\hat{\mathscr{X}}}=(\sigma^{pq}_{\hat{\mathscr{X}}})=\Gamma_L(\hat{\mathscr{X}},\hat{\mathscr{X}})$ and $\Delta_{\hat{\mathscr{X}}}=\det\sigma_{\hat{\mathscr{X}}}$ (we shall use similar notation for the other variables).

According to the truncation via $\hat{\psi}_{\varepsilon,\varepsilon'}$, we can now follow the argument in Yoshida [29, Section 4.2], *except that H's diffusion coefficient is replaced under our truncation*. This validates the conditional type Cramér condition: to be precise, for any $B>0$, distributional equivalence and the integration-by-parts formula yield that

$$\begin{aligned}
&E\left[\sup_{u:|u|\geqslant B}|E[\psi_{\varepsilon,\varepsilon'}e^{iuH_{t^0}}|X_0,X_{t^0}]|\right]\\
&\quad=E\left[\sup_{u:|u|\geqslant B}|E[\psi_{\varepsilon,\varepsilon'}e^{iu\tilde{H}_{t^0}}|X_0,X_{t^0}]|\right]\\
&\quad=\hat{E}\left[\sup_{u:|u|\geqslant B}|\hat{E}[\hat{\psi}_{\varepsilon,\varepsilon'}e^{iu\tilde{H}(t^0,\hat{v})}|X(t^0,\hat{v})]|\right]\\
&\quad=\hat{E}\left[\sup_{u:|u|\geqslant B}|(iu)^{-1}\hat{E}[e^{iu\tilde{H}(t^0,\hat{v})}\Psi(\hat{\psi}_{\varepsilon,\varepsilon'})|X(t^0,\hat{v})]|\right],
\end{aligned}\tag{35}$$

where $\mathscr{L}((\psi_{\varepsilon,\varepsilon'},\tilde{H}_{t^0})|P)=\mathscr{L}((\hat{\psi}_{\varepsilon,\varepsilon'},\tilde{H}(t^0,\hat{v}))|\hat{P})$, and the functional Ψ is given by

$$\begin{aligned}
\Psi(\hat{\psi}_{\varepsilon,\varepsilon'})&=\Gamma_L(X(t^0,\hat{v}),\sigma^{-1}_{X(t^0,\hat{v})}\hat{\psi}_{\varepsilon,\varepsilon'}\Gamma_L(X(t^0,\hat{v}),\tilde{H}(t^0,\hat{v})))\\
&\quad-\Gamma_L(\hat{\psi}_{\varepsilon,\varepsilon'},\tilde{H}(t^0,\hat{v}))-2\hat{\psi}_{\varepsilon,\varepsilon'}L\tilde{H}(t^0,\hat{v})\\
&\quad+2\sigma^{-1}_{X(t^0,\hat{v})}\hat{\psi}_{\varepsilon,\varepsilon'}\Gamma_L(X(t^0,\hat{v}),\tilde{H}(t^0,\hat{v}))LX(t^0,\hat{v})
\end{aligned}\tag{36}$$

which is well-defined on $\{\hat{\psi}_{\varepsilon,\varepsilon'}>0\}$. It follows from (35) that

$$E\left[\sup_{u:|u|\geqslant B}|E[\psi_{\varepsilon,\varepsilon'}e^{iuH_{t^0}}|X_0,X_{t^0}]|\right]\leqslant\frac{1}{B}\hat{E}[|\Psi(\hat{\psi}_{\varepsilon,\varepsilon'})|].$$

It suffices to show $\Psi(\hat{\psi}_{\varepsilon,\varepsilon'})\in L^1(\hat{P})$: if this is true, then (17) of [A3] follows by letting B be sufficiently large. Note that (18) in [A3] holds true with $\psi=\psi_{\varepsilon,\varepsilon'}$ since $P[\psi_{\varepsilon,\varepsilon'}>0]=\hat{P}[\hat{\psi}_{\varepsilon,\varepsilon'}>0]$ and this probability is positive by virtue of Lemma 6.

As remarked in Yoshida [29, Section 5.1], there exists a polynomial function $\mathscr{P}$ such that

$$|\Psi(\hat{\psi}_{\varepsilon,\varepsilon'})|\leqslant\mathscr{P}(1_{\{|\hat{\xi}_{\varepsilon,\varepsilon'}|\leqslant1\}}Q(t^0,\hat{v})^{-1},|U(t^0,\hat{v})|,|V(t^0,\hat{v})|,|U^*(t^0,\hat{v})|,|\sigma_{\hat{\xi}_{\varepsilon,\varepsilon'}}|),\tag{37}$$

where $Q(t^0,\hat{v}) = \det U(t^0,\hat{v})$, $V(t^0,\hat{v}) = L\hat{\mathscr{X}} \in \mathbb{R}^2$ and $U^*(t^0,\hat{v}) = \Gamma_L(U(t^0,\hat{v}), U(t^0,\hat{v})) \in \mathbb{R}^4 \otimes \mathbb{R}^4$.

(a) In Lemma 6, we have seen that $1_{\{|\hat{\xi}_{\varepsilon,\varepsilon'}|\leqslant 1\}} Q(t^0,\hat{v})^{-1} \in \bigcap_{p<\infty} L^p(\hat{P})$.

(b) Since $\hat{\mathscr{X}} \in D^L_{2,\infty-}$ and L takes its values in $\bigcap_{p<\infty} L^p(\hat{P})$, we see that $V(t^0,\hat{v})$ and $U(t^0,\hat{v})$ belong to $\bigcap_{p<\infty} L^p(\hat{P})$.

(c) Applying Theorems 10-3 and 10-17 of Bichteler et al. [8] repeatedly and then using Theorem 5-10 of the same monograph, it is not difficult to see that $U^*(t^0,\hat{v}) \in \bigcap_{p<\infty} L^p(\hat{P})$, taking into account that $\tau \in C_b^\infty(\mathbb{R}_+)$.

(d) Since $\hat{\xi}_{\varepsilon,\varepsilon'} \in \bigcap_{p<\infty} L^p(\hat{P})$, it follows from (34) and the property of L that $\sigma_{\hat{\xi}_{\varepsilon,\varepsilon'}} \in \bigcap_{p<\infty} L^p(\hat{P})$.

Summarizing the above now yields

Lemma 8. *Under Assumptions* 1 *and* 2, *we have* $\Psi(\hat{\psi}_{\varepsilon,\varepsilon'}) \in L^1(\hat{P})$ *for* Ψ *of* (36).

Combining Lemmas 6, 7 and 8 guarantees $[A3^Q]$ of Yoshida [29], therefore the proof of Theorem is complete.

Acknowledgements

The authors thank the referee for his careful reading of the manuscript and valuable comments. They are also grateful to the editor for his encouragement.

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