FLAT CONNECTIONS, BRAID GROUPS AND QUANTUM GROUPS

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I will begin by reviewing the construction of a flat connection ∇ on the Cartan subalgebra of a complex, simple Lie algebra $\mathfrak{g}$ with simple poles on the root hyperplanes and values in any finite-dimensional $\mathfrak{g}$–module V. This connection, which was obtained in joint work with J. Millson, is a generalisation of the (genus 0) KZ connection to configuration spaces of other Lie types and of Cherednik's rational Dunkl operators for the Weyl group W of $\mathfrak{g}$. Its monodromy gives a one–parameter family of representations of the generalised braid group B_W of type W deforming the action of the (Tits extension of) W on V.

I will then explain how the work of Drinfeld and Kohno on the KZ connection leads one to conjecture that the monodromy of ∇ is described by Lusztig's quantum Weyl group operators and sketch the recent proof of this conjecture. One of its key ingredients is the novel notion of *quasi–Coxeter algebras*, which are to Brieskorn and Saito's Artin groups what Drinfeld's quasi–triangular, quasi–bialgebras are to the classical braid groups. Time permitting, I will motivate their definition by using De Concini and Procesi's compactifications of hyperplane complements which yields, in the case of the Coxeter arrangement of type A_{n-1}, the moduli space $\overline{\mathcal{M}}_{0,n+1}$ of stable, $n+1$–marked curves of genus zero.

I will also describe the semi–classical analogue of this conjecture which, through the work of Boalch, relates the De Concini–Kac–Procesi action of B_W on the Poisson–Lie group G^* to the isomonodromic deformation of G–connections on the punctured disk having a pole of order 2 at the origin.