Title: "Overview with elementary definitions and examples"

May 13 (Mon.) 15:00 - 16:00, 16:20 - 17:20

Abstract:

The "Fundamental Theorem of Combinatorial Group Theory" says that a word on X represents the trivial element of a group $G = \langle X|\Delta\rangle$ if it is the product of edge labels on a *cancellation diagram* which is a planar graph with faces labelled with elements of $\Delta \coprod \Delta^{-1}$. In parametrized Morse theory we are lead in a natural way to consider the dual of a cancellation diagram which some call a *spherical diagram* but I call a *picture for* $H_3(G)$ since it represents an element of $H_3(G, \mathbb{Z})$. These are planar graphs in which the vertices are labelled with relations ($\in \Delta^{\pm}$). The group is the *Steinberg group* on the integer group ring $\mathbb{Z}[\pi]$ of the fundamental group $\pi = \pi_1 M$ of the manifold on which we are doing Morse theory.

One of the first successful attempts at computing the higher Franz-Reidemeister torsion arose from this idea applied to the very elementary case of circle bundles over S^2. Using a unitary representation of the fundamental group of the total space E, we obtain a picture for the Steinberg group of the complex numbers. However, by an old theorem of Gersten we have:

$$H_3(St(R)) = K_3 R$$

for any ring R. So we are looking at the third *algebraic K-theory* group of the complex numbers $\mathbb{C}$. The *Bloch-Wigner dilogarithm* gives a homomorphism

$$D : K_3\mathbb{C} \to \mathbb{R}.$$

To generalize this to higher dimensions we are lead to higher dimensional analogues of pictures and their duals and to the higher algebraic K-theory of the complex numbers. I use two definitions for higher algebraic: *Volodin K-theory* and *Waldhausen K-Theory.*

In this first lecture I will explain the Volodin definition and its relation to Morse theory. I will also go through the definitions of cancellation diagrams and pictures explaining in detail everything that I said above with some simple and not so simple examples.

References:

Lyndon, R and Schupp, P, *Combinatorial Group Theory*, Springer CIM, 1977.

Cynthia Hog-Angeloni, Wolfgang Metzler, Allan J. Sieradski (ed), *Two-Dimensional Homotopy and Combinatorial Group Theory*, LMS Lecture Notes, Cambridge University Press, 1993.

Igusa, K and Klein, J, *The Borel regulator map on pictures II. An example from Morse theory*, K-Theory 7 (1993), no.3, 225-267.